Danmarks Matematiklærerforening

School Mathematics Seen Internationally

Jeremy Kilpatrick
University of Georgia

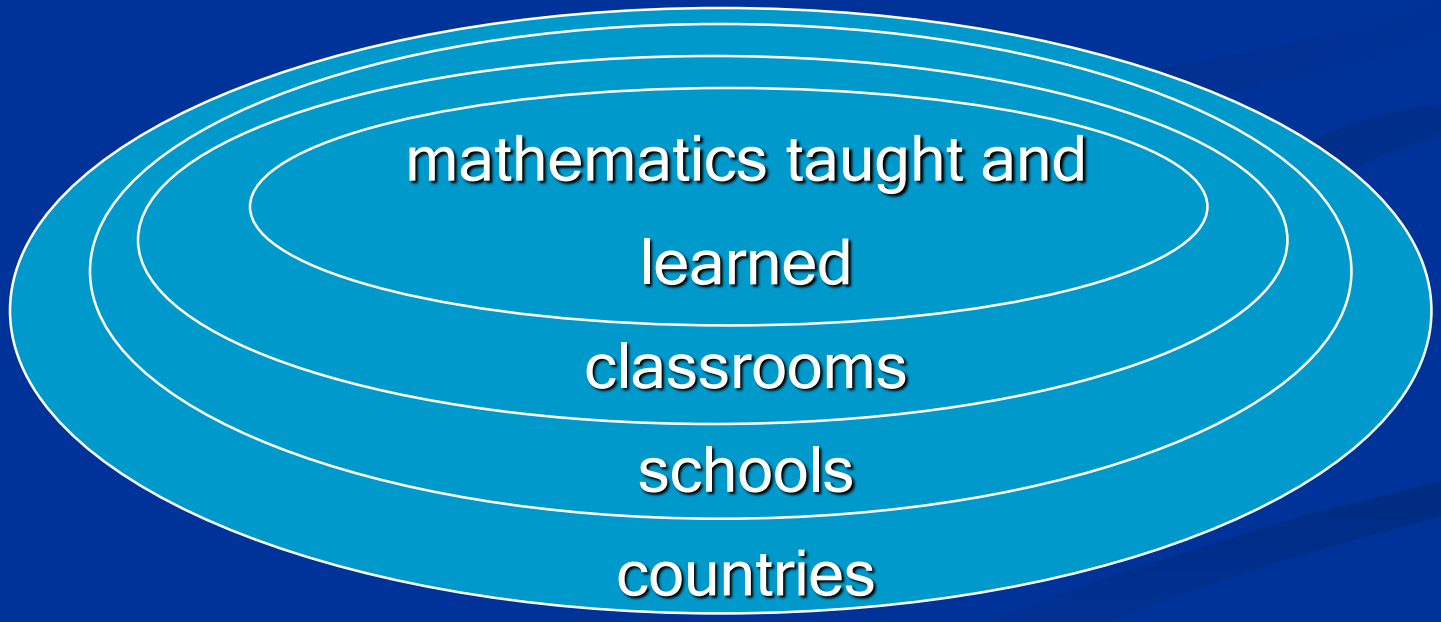
Outline

- The space of school mathematics
  - Content versus teaching
  - Universal versus local
- School mathematics content is the same everywhere
- School mathematics teaching is the same everywhere
- School mathematics content is different everywhere
- School mathematics teaching is different everywhere
- Where are we headed?

The space of school mathematics

Mathematics education – a complex of systems

From the classroom to the country, each system has structural units aligned, at national and local levels, with political units



We can view school mathematics as a hierarchy - decisions at the top filter down to classrooms

- *More accurately, systems are interlocking and interpenetrating*
- *Vector of change:* ⇓ ⇑ ⇒
- *Like the ocean:*

- One should not assume that strong centralized control of school mathematics will enable curriculum change to come from the top down
- Despite claims to the contrary, school systems are very much alike
- Always a gap between official pronouncements and actual practice (e.g., France and England)

Centralised systems are not so centralised and decentralised systems are not so decentralised, as commonly supposed. As a French school inspector once observed: ‘In France, every teacher is supposed to be doing the same thing but nobody is, and in England, where everyone is supposed to be going his own way, nobody is’

James Koerner, quoted in G. Howson, C. Keitel, & J. Kilpatrick, *Curriculum Development in Mathematics,* 1981

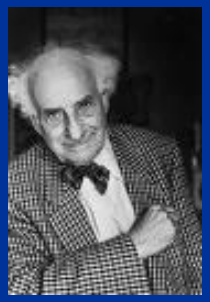
- Any survey of the field of mathematics education internationally is necessarily incomplete
- The field has grown so much over the past century that it has become impossible to survey adequately even the most recent developments, let alone get a good perspective on its growth over the past few decades

- This talk, therefore, cannot be comprehensive
- At best, I can offer only a few glimpses of what seems to me to be going on in school mathematics seen internationally
- To make the task manageable, I have partitioned the space of school mathematics:
  - content versus teaching, and
  - universal versus local

Content versus teaching

Content versus teaching

- Content: *What* is taught
- Process: *How* it is taught
- Hans Freudenthal: You cannot separate the two:



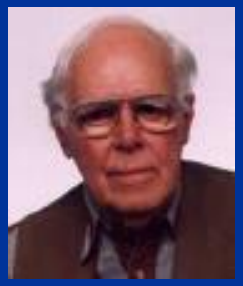
  - "General teaching theory is no science at all but an empty form the filling of which is a phantom. There is no instruction without content and no science of instruction without content." (Freudenthal, 1978, p. 163)

- Nonetheless, I look at these two aspects of school mathematics separately

Universal versus local

Universal versus local

- From a distance, mathematics is often seen as the school subject that is most universal
- Martin Gardner:

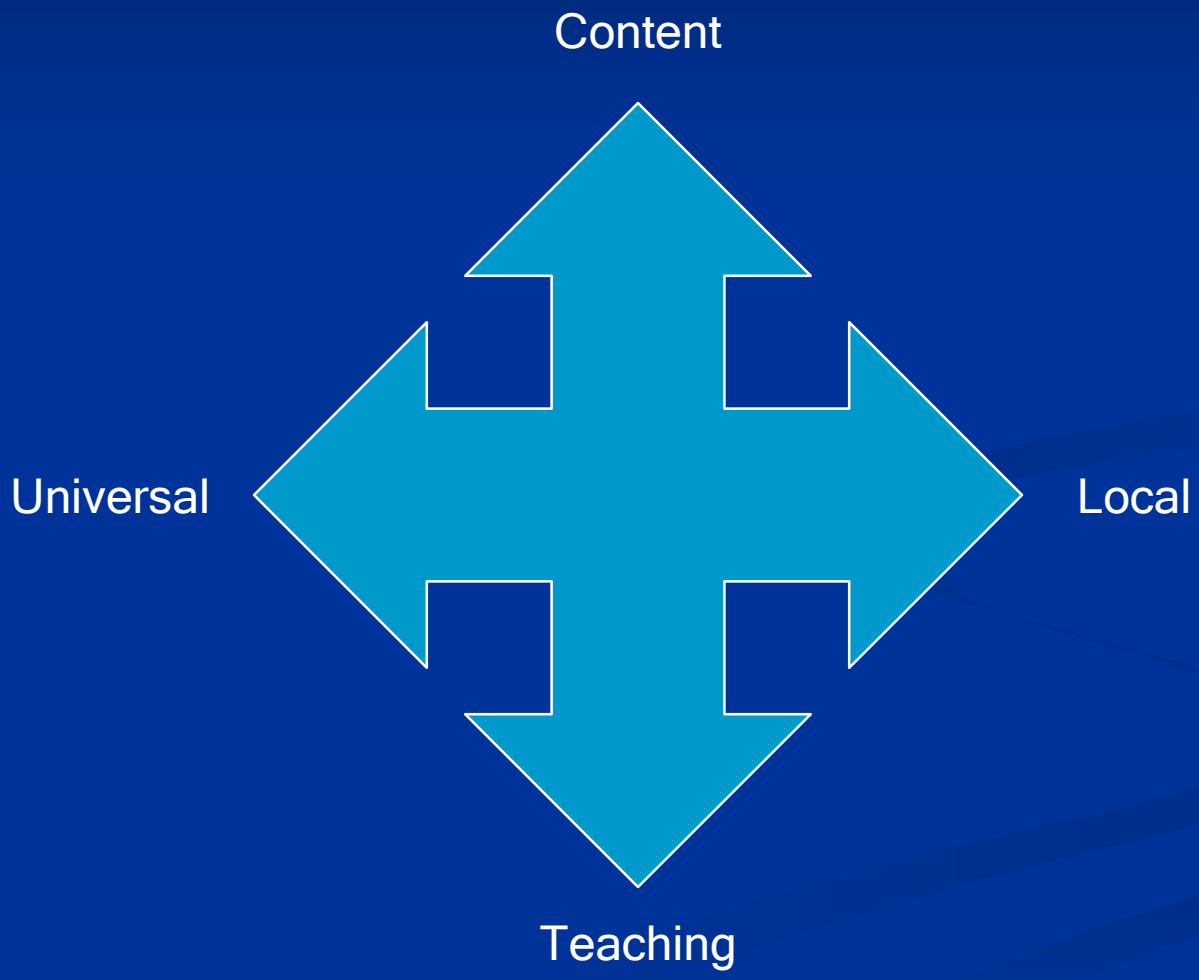


  - "One must keep in mind that mathematics, like science, is a cumulative process that advances steadily by uncovering truths that are everywhere the same" (Gardner, 1998, p. 9)
- Up close, school mathematics is embedded in history and culture:
  - Language used
  - Applications drawn from familiar cultural practices

Cross the dimensions

- Cross the dimensions
- Then take extremes
- Four claims used to structure what follows:
  - Content is universal
  - Teaching is universal
  - Content is local
  - Teaching is local

The space of school mathematics



School mathematics content is the same everywhere

Same content

- Relatively stable structure:
  - Primary grades
    - Computational arithmetic
    - Measurement & informal geometry
  - Middle grades
    - Algebra & more informal geometry
  - Secondary school
    - Algebra elaborated and formalized
    - Rational, exponential, and trigonometric functions
    - Preparation for calculus
  - Geometry, probability, and statistics less firmly anchored

"Canonical" curriculum

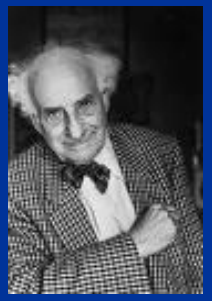
- Geoffrey Howson & Brian Wilson (1986)
  - *School Mathematics in the 1990s*
- Large-scale studies comparing pupils' achievement across countries assume a common curriculum:
  - First International Study of Mathematics
  - Trends in International Mathematics and Science (TIMSS) studies
  - Programme for International Student Achievement (PISA) studies
    - (For more information about these studies, see Niss, Emanuelsson, & Nyström, 2013)

Large-scale international comparative studies

- Make use of a framework to characterize mathematical content
- Measure pupils' achievement by test items developed to fit that framework
- Result: a kind of idealized curriculum that matches no country's curriculum exactly (Keitel & Kilpatrick, 1999)
- In other words, such studies assume that school mathematics content is essentially the same around the world

International Association for the Evaluation of Educational Achievement (IEA)

- Studies using *opportunity to learn (OTL):*
  - First and Second International Studies of Mathematics
  - Trends in International Mathematics and Science Studies (TIMSS)
- Teachers estimate percent of pupils who had opportunity to learn content of test question
- Hans Freudenthal (1975) strongly criticized OTL:



  - IEA assumed OTL based on curriculum programs
  - Actually based on IEA tests

Measuring instrument becomes goal

- United States
  - States, districts, and consortia participating in TIMSS "benchmarking" studies
  - TIMSS frameworks adopted by various states and districts
- Singapore
  - Fifth International Conference on Science and Mathematics Education (CoSMEd 2013) in Penang, Malaysia
  - Singapore colleague reported that they were revising curriculum using TIMSS and PISA test questions on which Singapore pupils did not do well

School mathematics teaching is the same everywhere

“Conventional” teaching

- Stereotype of current mathematics teaching (a teacher-centered transmission model):
  - Teacher at front of classroom demonstrating the right way to do problems
  - Pupils expected to memorize facts, follow rules, and learn procedures
- Stereotype has large grain of truth
  - Survey of secondary mathematics teachers and pupils in England (Pampaka et al., 2012)

Survey by Pampaka et al.

- Both teachers and pupils tended to characterize teaching as “transmissionist” and not “connectionist”
- Teachers caught between
  - Encouragement by professional organizations to engage in connectionist (learner centered) teaching
  - Performance management system focuses on grades and encourages teaching to the test
- Conflict makes change risky
- Framework documents in England (like Common Core State Standards in U.S.) likely to reinforce teacher-centered instruction

School mathematics content is different everywhere

Video Studies and Textbooks

- A few videotapes enough to show content differences, which can also be seen in textbooks
- Vilma Mesa (2009) analyzed conceptions of function in 7th- and 8th-grade textbooks
  - 35 textbooks from 18 countries in TIMSS (1995)
  - Four clusters: rule oriented, abstract oriented, abstract oriented with applications, and applications oriented
  - Found no canonical curriculum for teaching function
  - Variation both within and across countries
- Measurement a site for local considerations
  - Germany: *Pfund* (metric pound, 500 grams)
  - Denmark: *Pund*

Measurement

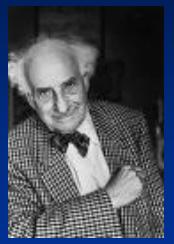
- Pupils learn and use own society's measurements
- Mathematics Forum for PISA 2003 (Berlin)
  - Linear equation to convert Fahrenheit to Celsius using freezing and boiling points of each scale
  - E.g., you land in Atlanta and hear the temperature is 95 degrees
  - Other forum participants thought item was not reasonable
  - Who converts? (outside the U.S.)
  - Not reasonable or not in curriculum?
- Many measurement issues linked to country's culture; e.g., legacy units of measure
  - Denmark: *Pund, fod,* and *dusin*

School mathematics teaching is different everywhere

Video Studies

- 1995 TIMSS Video Study (Stigler & Hiebert, 1999)
  - 8th-grade classrooms in Germany, Japan, and the U.S.
  - Hypothesis: Lessons within countries tend to show greater similarity than across countries (e.g., reviewing, introducing)
  - Perhaps a cultural "script" for teaching mathematics
- 1999 TIMSS Video Study (Hiebert et al., 2003)
  - 8th-grade classrooms in Australia, the Czech Republic, Hong Kong, Japan, the Netherlands, Switzerland, and the U.S.
  - Consistent with hypothesis but not uniform script
  - E.g., Switzerland had much variation across lesson structure
    - Language differences
    - Reform activities promoting two methods of teaching

Efforts to Move Away from Transmission



- Realistic Mathematics Education (RME)
  - Emerged from work of Freudenthal and his Institute
  - Children should not be confronted with ready-made mathematics
  - Mathematizing reality
  - RME has spread from the Netherlands to countries such as Brazil, Denmark, England, Germany, Malaysia, Japan, Portugal, South Africa, Spain, and the U.S.
- PRIMAS Consortium (2010-2013)
  - Promote inquiry-based learning (IBL) in mathematics & science education across Europe
  - Included 14 universities from 12 European countries
  - In Denmark, based at Roskilde U. and directed by Morten Blomhøj
  - Recognized that IBL understood and implemented differently

Where are we headed?

Forces Motivating Change

- Attract pupils
  - "The central motivation for the PRIMAS project with respect to mathematics was a desire to increase the number of pupils who continue their study of mathematics and ultimately seek to be employed in a mathematics-related field" (Schoenfeld & Kilpatrick, 2013, p. 904)
  - PRIMAS was established in effort to attract more pupils to study of mathematics; such efforts likely to continue as countries deal with shortages of mathematically trained workers–in part because of how mathematics is being taught
  - PRIMAS and RME make use of realistic applications of mathematics
  - Denmark's tradition of attention to modeling spreading elsewhere
  - Collection on similar curriculum efforts in various countries (Li & Lappan, 2014)

Forces Motivating Change

- Technology
  - Technology likely to produce changes in school mathematics
    - In what is taught
    - In how it is taught
  - Course content certain to change as pupils use computing tools to explore realistic problems that they could never have approached before
  - Technology changes will also allow pupils to learn more mathematics interactively online than ever before
  - It is impossible to predict how technology will affect the school mathematics curriculum in any country or school
  - But it clearly will

A Force Hampering Change

- Assessment
  - Recent handbook chapter entitled “Toward an International Mathematics Curriculum” by Jinfa Cai and Geoffrey Howson (2013)
    - Address increasing tendency for countries to include internationally accepted set of core topics in national curriculum
    - See tendency as something to be welcomed and accepted as well as a potential obstacle to curriculum development
  - They point out that the heavy use of public examinations in such countries as Denmark, England, China, and the United States, for example, poses particular threat to curriculum changes such as the incorporation of mathematical modeling into the curriculum

A Force Hampering Change

- Cai and Howson (2013) point out:
  - What is considered the "common core" of an international school mathematics curriculum cannot possibly constitute a satisfactory curriculum in itself
  - Individual countries need to be the seedbeds for new initiatives
- "Curriculum development aimed at other ends than simply improving the teaching and learning of accepted curriculum content–although that in itself is a worthy aim–depends upon the freedom of individual countries to experiment within the confines of their curriculum. The absence of that freedom, through the emergence of an all-embracing international curriculum, would seriously prevent any future developments in the curriculum as a whole" (p. 969)

What Is Right?

- Married couple went to rabbi to help resolve a quarrel
- Wife went in and told her story, whereupon rabbi nodded and said, “You are right”
- Husband went in and told his side of the story to rabbi, who nodded and said, “You are right”
- After couple had gone on their way, rabbi’s wife, who had overheard both exchanges, said to her husband, “They cannot both be right”
- The rabbi nodded and said, “You are right”
- I have proposed four conflicting claims about school mathematics
- They cannot all be right
- From your angle of vision, at least one of them may be right for you