Developing Children's Mathematical Imagination as a Key to Helping Them Learn:

Mathematics Learning for 2030 Life

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Focus for this talk

- Learning mathematics for 2030
- Exploring the potential of Open Tasks and investigations for mathematics learning
- How do we learn mathematics? – the role of the mathematical imagination as a missing link for many students.
- The role of the imagination, creativity, gestures and talk when learning mathematics

I touch the future… I teach.

"I touch the future…I teach." Christa McAuliffe

Ref: https://divinesparkignites.com/2015/01/29/i-touch-the-future-i-teach-christa-mcauliffe/

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5 predictions for what life will be like in 2030

Next stop ... driverless cars, smart homes and genetically engineered pets

Image: REUTERS/Tyrone Siu

31 Oct 2017

Mike Moradi

This article is part of the Annual Meeting of the Global Future Councils

Ref: https://www.weforum.org/agenda/2017/10/tech-life-predictions-for-2030/

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Photos of Gus Gervasoni

pwc

Ref: Price Water Coopers (2018). *Workforce of the future*, p. 4. Retrieved: https://www.pwc.com/gx/en/services/people-organisation/workforce-of-the-future/workforce-of-the-future-the-competing-forces-shaping-2030-pwc.pdf

www.pwc.com/people

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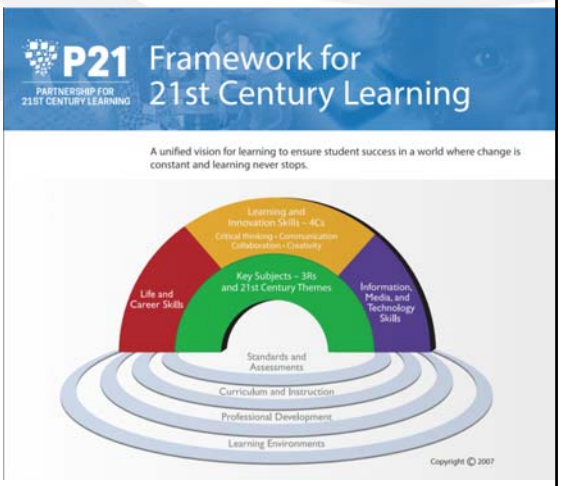
Learning and Innovation Skills

Learning and innovation skills are what separate students who are prepared for increasingly complex life and work environments in today's world and those who are not. They include:

- Creativity and Innovation
- Critical Thinking and Problem Solving
- Communication
- Collaboration



Reference:
Partnership for 21st Century Learning (2017). Retrieved: http://www.p21.org/storage/documents/docs/P21_framework_0816.pdf

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21st century skills

Preparing students for a changing world

Young people are preparing for a very different world from the one we know.

Young Queenslanders in the 21st century need to be

- Innovators
- Entrepreneurs
- Lifelong learners
- Responsible global citizens

What are the 21st century skills in the General senior syllabuses?

Critical thinking
- analytical thinking
- problem-solving
- decision-making
- reasoning
- reflecting and evaluating
- intellectual flexibility

Creative thinking
- innovation
- initiative and enterprise
- curiosity and imagination
- creativity
- generating and applying new ideas
- identifying alternatives
- seeing or making new links

Communication
- effective oral and written communication
- using language, symbols and texts
- communicating ideas effectively with diverse audiences

Collaboration and teamwork
- relating to others (interacting with others)
- recognising and using diverse perspectives
- participating and contributing
- community connections

Personal and social skills
- adaptability/flexibility
- management (self, career, time, planning and organising)
- character (resilience, mindfulness, open- and fair-mindedness, self-awareness)
- leadership
- citizenship
- cultural awareness
- ethical (and moral) understanding

ICT skills
- operations and concepts
- accessing and analysing information
- being productive users of technology
- digital citizenship (being safe, positive and responsible online)

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

For all Queensland schools

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Ref: Queensland Curriculum and Assessment Authority. (2018). Retrieved: https://www.qcaa.qld.edu.au/downloads/senior/snr_syll_redev_21st_century_skills_preparing_students.pdf

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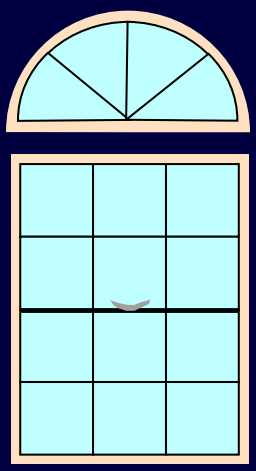

You are looking through the window into a mathematics classroom that embraces 21st century skills to prepare children for using mathematics in their adult life in 2030.

What would the classroom be like?
What would you see, hear, feel?

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The Australian Curriculum

Mathematics aims to be relevant and applicable to the 21st century. The inclusion of the proficiencies of understanding, fluency, problem-solving and reasoning in the curriculum is to ensure that student learning and student independence are at the centre of the curriculum. …These proficiencies enable students to respond to familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems efficiently.

Ref: https://www.australiancurriculum.edu.au/resources/mathematics-proficiencies/

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Our Challenge as School Mathematics Leaders

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- How do design mathematics classrooms that provide experience of the skills we predict are needed in 2030?
  - Creativity and innovation
  - Critical thinking and problem solving
  - Communication
  - Collaboration

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Open Tasks – Learning Mathematics for the 21st Century

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Geometry Open Task

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Open Task for Investigation	
Imagine that you have 10 long and 10 short sticks to make polygons for a mobile. Represent the shapes hanging from the finished mobile.	

1. Create as many solutions as possible, including some creative challenging examples or some new thinking.
2. You learn best by asking questions and giving explanations as you work on the solutions.

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Discussing Solutions for Open Task

In groups of two or three:
Share and explain your solutions and identify and discuss your most creative/new thinking solution

Class Plenary. Discuss 2-3 solutions that raise the level of mathematical thinking, reasoning and possibilities.

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7 Practices for Orchestrating Productive Discussions

A. **Anticipating** potential student responses
B. **Monitoring** student responses interactively
C. **Selecting** representative responses for later presentation
D. **Sequencing** student responses
E. **Presenting** student work so that other students can see it
F. **Encouraging** students to learn by listening to other students
G. **Connecting** the students' strategies with the formal processes that were the intention of the task in the first place

Adapted from Smith, M. S., & Stein, M. K. (2011).

Ref: Australian Academy of Science (2018). Mathematics by inquiry. Retrieved: https://www.resolve.edu.au/years-f-6-assessing-mathematical-reasoning

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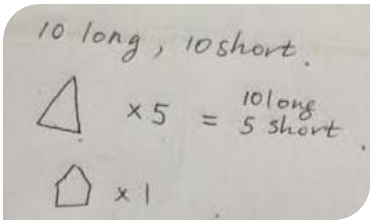

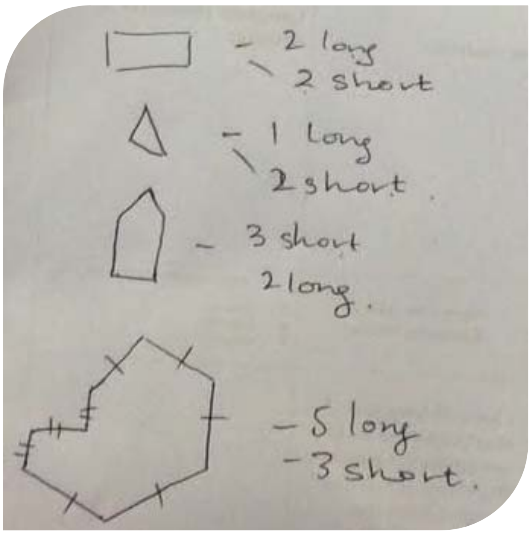

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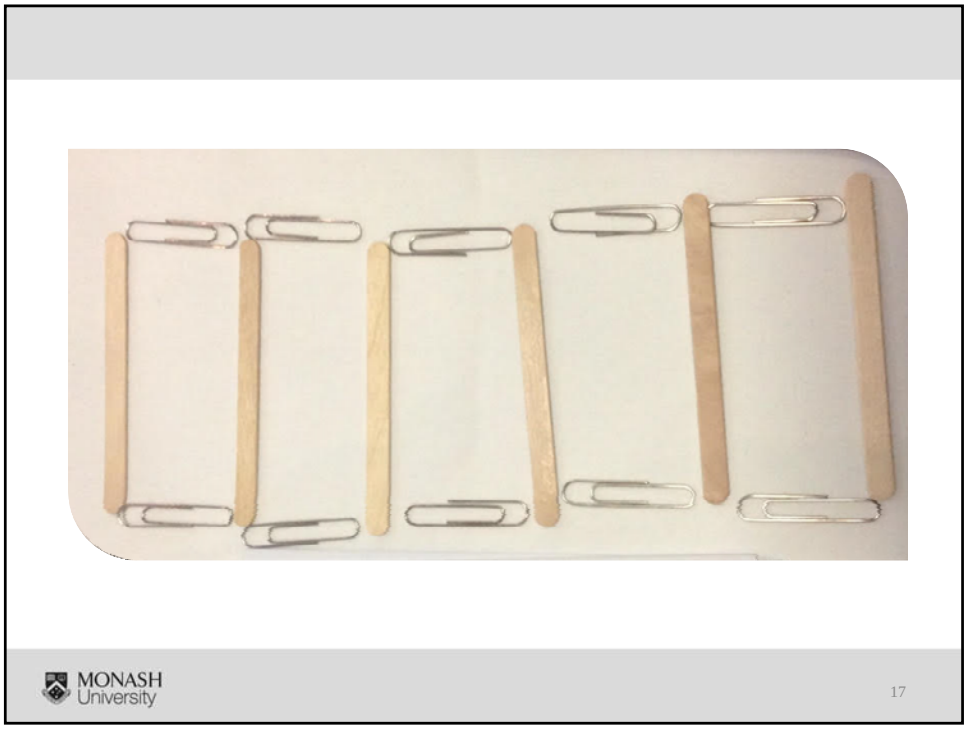
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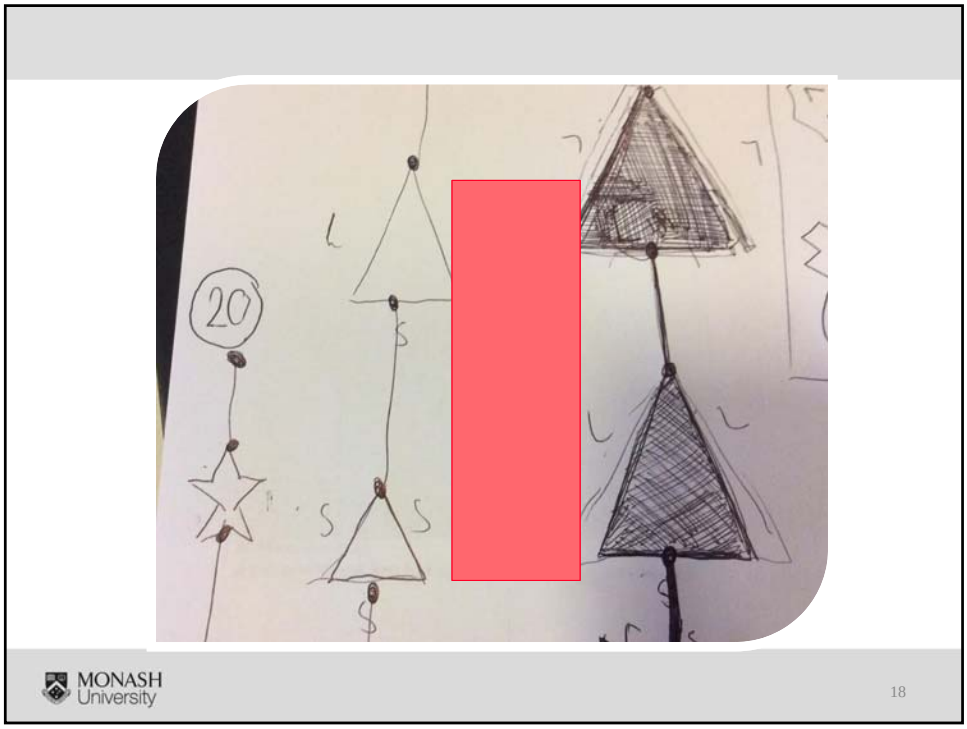
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Australian Curriculum: Reasoning

Students develop an increasingly sophisticated capacity for logical thought and actions, such as **analysing**, **proving**, **evaluating**, **explaining**, **inferring**, **justifying** and **generalising**. Students are reasoning mathematically when they **explain** their thinking, when they **deduce** and **justify** strategies used and conclusions reached, when they **adapt** the known to the unknown, when they **transfer** learning from one context to another, when they **prove** that something is true or false and when they **compare** and **contrast** related ideas and **explain** their choices.

Australian Curriculum Assessment and Reporting Authority (2017).

MATHEMATICAL REASONING PROMPTS

ANALYSING

- What is the same and different about ...?
- What stays the same and what changes?
- Sort or organise the following according to ...
- Alter an aspect of something to see an effect. If we change this what will happen?
- What follows from this? What do you think will happen next if we do this?
- What do you notice...?
- When is it true?
- Is it just sometimes true, or is it always true?

GENERALISING

- How can you describe what is the same?
- What is the rule?
- What is the pattern here?
- How can you describe the pattern?
- What happens in general?
- Is that ... (pattern) always going to work?
- Are there other examples that fit the rule?
- How could you explain the rule to someone else?

JUSTIFYING

- Is this conjecture just sometimes true, or always true?
- How do you know?
- How could we show or prove that it is true?
- True or false? Why? Let's justify.
- Convince me...
- How can we be sure...?
- Tell me what is wrong with....
- Explain - why does this (process/rule/result) work?
- Can you go through that step by step?
- Can you explain that step by step?
- Why?
- If...then...

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Open Tasks are useful for mathematics teaching in that they:

- allow students to investigate, make decisions, generalise, seek patterns and connections, justify, communicate, discuss, and identify alternatives;
- focus students' attention onto key aspects of concepts, as distinct from them merely trying to recall a rule or procedure;
- are able to be used with classes of varying abilities since students can approach the tasks at different levels and in different ways – personalised learning/precise with teaching;
- allow students more opportunities for creative thinking than conventional tasks; and
- address content explicitly and so make it clear to students what it is intended they learn;
- Important opportunities for teachers to assess children's mathematical thinking and reasoning.

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Approaching Open Tasks

- We need to remind students that their challenge is to work out a range of solutions, including some creative examples and new thinking.
- We need to emphasise that students need to challenge themselves to do some hard thinking, and some creative thinking when working out a set of solutions.
- We need to emphasise the way that discussion contributes to learning for the students, and the way in which students can benefit from hearing the perspectives of their peers.
- We need to emphasise that you learn from having a go and making mistakes. Celebrate this!!

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Ref: Peter Sullivan 2003

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A lesson structures based on an open task

Prior to the lesson

1. Work through the task and identify a range of solutions.
2. Identify the potential 'difficult' aspects for some students.
3. Plan a series of enabling and extending prompts.

Lesson Stages

1. Pose the open task- LAUNCH
2. Make the mathematics and style of learning explicit.
   - Aim to work out as many solutions as possible in the time frame
   - People learn mathematics effectively when they ask questions and explain their solutions and strategies
3. Students work on the open task – EXPLORE (while the teacher roves, observes and provides enabling and extending prompts)
4. Whole group review – Discussion and Reflection
   - Teacher selects solutions for students to discuss that highlight the key mathematical ideas and raise the level of thinking
5. Teacher summary of the key mathematical ideas

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Adapted from Peter Sullivan (2016)

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Geometry Open Task

Open Task for Investigation	Enabling Prompt	Extending Prompt
Imagine that you have 10 long and 10 short sticks to make polygons for a mobile. Represent the shapes hanging from the finished mobile.	You have 10 long and 10 short straws to make pentagons for a mobile. Represent the shapes hanging from the finished mobile.	You have 24 long and 24 short straws to make polygons for a mobile. Represent the shapes hanging from the finished mobile.

1. Create as many solutions as possible, including some creative challenging examples or some new thinking.
2. You learn best by asking questions and giving explanations as you work on the solutions.

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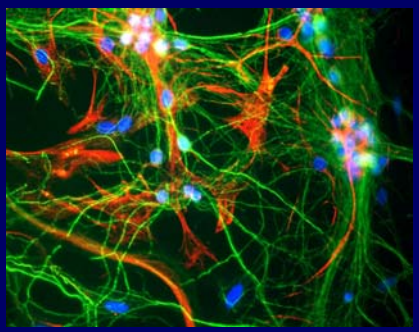
What happens in the brain when you learn mathematics?

Neuroplasticity and Learning

- The brain is constantly changing.
- Every experience we have fires neural pathways.
- These neural pathways are constantly being established or removed.



Rat cortical neurons and glia in mixed tissue culture (green and red) and nuclei of all cells stained (blue).

REF:
http://www.encorbio.com/Album/pages/ChkMAP2-GFAP-Hoe-40X-2.htm

PERCEPTUAL & MOTOR EXPERIEMCES
Touching
Seeing
Hearing
Smelling
Tasting
Moving

SIMULATING
Touching
Seeing
Hearing
Smelling
Tasting
Moving

LANGUAGE – Talking, describing, explaining

Brain Cortex

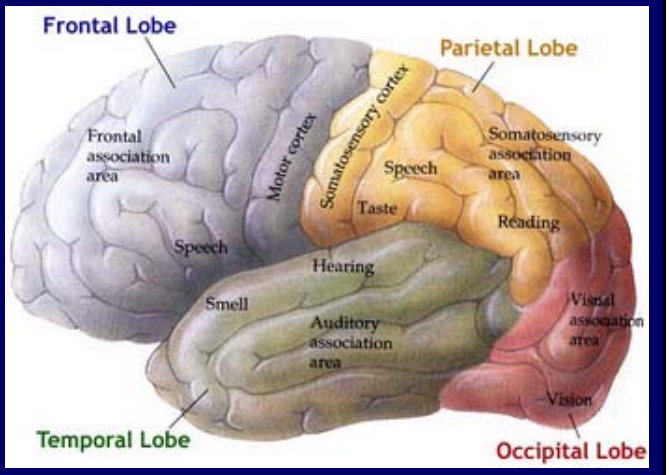


Image from http://i7.photobucket.com/albums/y291/shoty2010/BrainLobesandSenses.bmp~original

PERCEPTUAL & MOTOR EXPERIEMCES
Touching
Seeing
Hearing
Smelling
Tasting
Moving

SIMULATING
Touching
Seeing
Hearing
Smelling
Tasting
Moving

LANGUAGE – Talking, describing, explaining

Learning

Enactive/Motor & Perceptual Systems

↓

Iconic/Simulation – Mental Image

↓

Abstract – Language/symbols

Handfuls!

1. **Take a handful of counters, and then predict how many counters are in your own hand.**
2. **Organise the counters so that anyone walking past can work out how many there are by looking and thinking (without counting).**

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Quantifying by organising, looking and thinking (not counting…)

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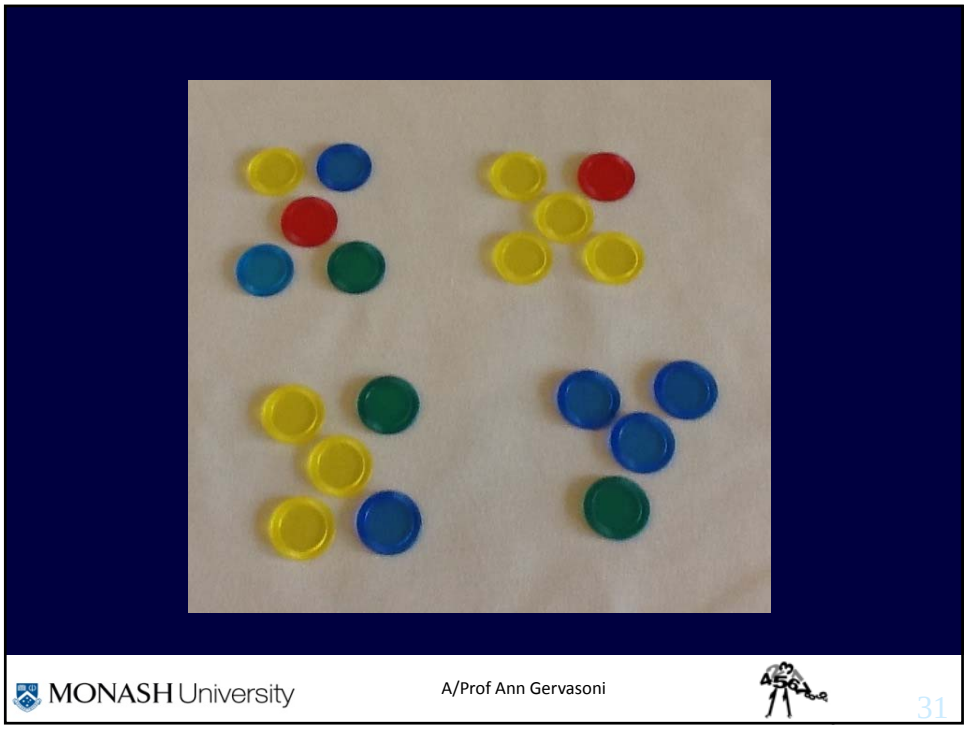
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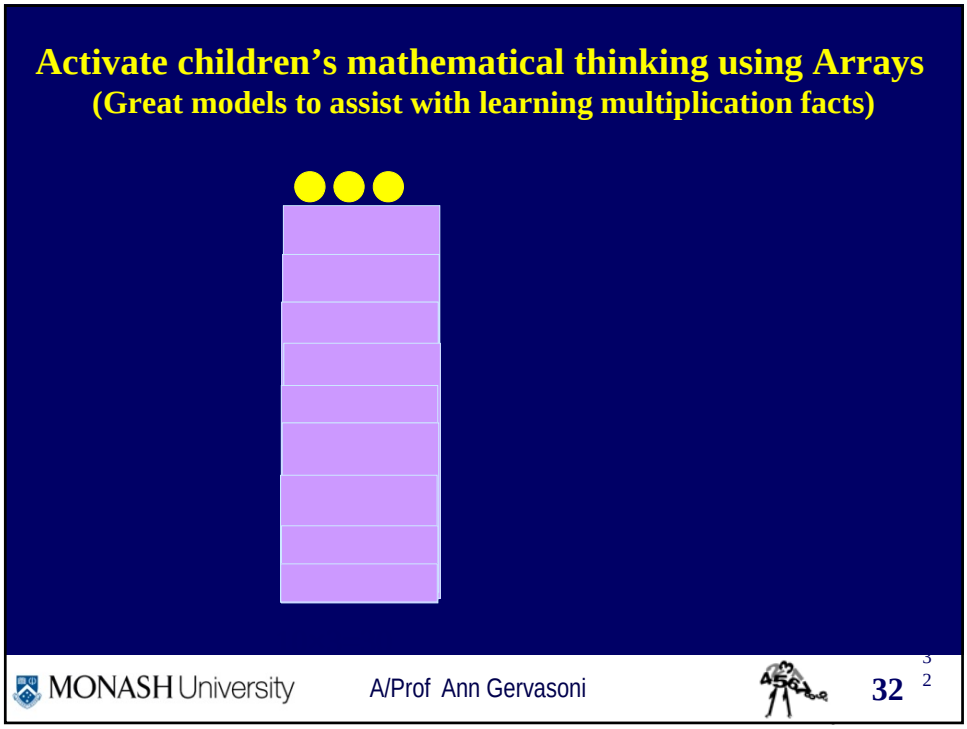
Activate children's mathematical thinking using Arrays
(Great models to assist with learning multiplication facts)
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Exploring Arrays *Open Task*

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Open Task for Investigation	Enabling Prompt	Extending Prompt
Imagine that you organised a handful of tokens into an array on the table (with all the tokens used). For each solution, show (1) what the array could be, and (2) the total quantity of tokens in each handful.		

1. Create as many solutions as possible, including some creative challenging examples.
2. You learn best by asking questions and giving explanations as you work on the solutions.

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Challenge Faced By Teachers Due To Student Diversity

42 classroom teachers responded to the survey prompt

"Describe the greatest challenge you face in the teaching of mathematics"

The most common theme emerging (noted by 14 teachers) was *the challenge of catering for a wide range of abilities in one classroom*.

Illustrative excerpts include:

- ***Having so many children in the classroom at different maths levels;***
- ***Catering for the huge range of maths knowledge within my classroom; and***
- ***Structuring lessons to differentiate to the various growth points.***

Gervasoni,A. & Roche, A. (2018). *Insights about how to create flexible mathematics classrooms that enable all students to thrive.* In Hsieh, F.-J. (Ed.), (2018). *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*, Vol 2, Taipei, Taiwan: EARCOME., pp 388-397.

Changes in Teaching Practices 12 months later…

Classroom teachers were asked to describe the *greatest change in their practice* as a result of their school's recent focus on the teaching and learning of mathematics.

Twenty (of 52 comments) involved *catering for diversity*.

The greatest change has been the way in which I approach the different students in my class.

That all students can access the same maths task at their own level and the importance of open-ended maths questions [for enabling this differentiation]

Continually checking what children understand, to assist planning.

Gervasoni,A. & Roche, A. (2018). *Insights about how to create flexible mathematics classrooms that enable all students to thrive.* In Hsieh, F.-J. (Ed.), (2018). *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*, Vol 2, Taipei, Taiwan: EARCOME., pp 388-397.

Challenge, Hard Thinking, Collaboration

"How can maths lessons be improved at your school?"

One theme that emerged from analysis of the survey responses from 57 students was the importance of *challenge and hard thinking.*

Students (8) indicated they would prefer more challenge, e.g.,

I would like to be challenged more.

I would like more challenges.

There could be more tricky questions. More advanced things for me to do.

Gervasoni,A. & Roche, A. (2018). *Insights about how to create flexible mathematics classrooms that enable all students to thrive.* In Hsieh, F.-J. (Ed.), (2018). *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*, Vol 2, Taipei, Taiwan: EARCOME., pp 388-397.

EMU students' perceptions about learning mathematics

Statements about learning mathematics	***Mean /5***	***% agree (4 or 5)***
I like to think hard in maths (***n***=30)	4.6	93%
I like to explain my thinking when doing maths (***n***=29)	4.2	86%
I like working with others when doing maths (***n***=30)	4.3	80%
Making mistakes in maths helps me to learn (***n***=30)	4.4	87%
I feel confident learning maths in the classroom (***n***=29)	4.1	79%
I feel confident learning maths at home (***n***=28)	4.2	82%

Gervasoni,A. & Roche, A. (2018). *Insights about how to create flexible mathematics classrooms that enable all students to thrive.* In Hsieh, F.-J. (Ed.), (2018). *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*, Vol 2, Taipei, Taiwan: EARCOME., pp 388-397.

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EMU specialist teachers' perceptions of EMU students' change in dispositions for learning mathematics in respect to challenge, effort, perseverance and explanations (n=121)

Statements about learning mathematics	Mean prior to EMU	Mean now	Change in mean
Students are confident to share their thinking and strategies	3.2	8.3	5.1
Students extend and challenge themselves	2.9	7.8	4.9
Students believe they can achieve in mathematics with effort	3.3	8.2	4.9
Students persevere even if the task is challenging	2.8	7.7	4.9

Gervasoni,A. & Roche, A. (2018). *Insights about how to create flexible mathematics classrooms that enable all students to thrive.* In Hsieh, F.-J. (Ed.), (2018). *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*, Vol 2, Taipei, Taiwan: EARCOME., pp 388-397.

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Challenge and Hard Thinking

- **Many teachers and students valued lessons that promoted mathematical challenge, hard thinking, persistence, explaining their mathematical thinking to others, and collaboration.**
- **This was also true for the most mathematically vulnerable 6-y-o students who participated in an EMU intervention program.**
- **These dispositions and skills are well aligned to predictions about the approaches to mathematics learning necessary for the future.**

Valuing challenge and hard thinking during mathematics lessons

School leaders were interviewed and asked to describe ways in which their focus on transforming mathematics teaching and learning in their school had impacted on their students. They reported that students enjoyed the challenge and hard thinking now required during mathematics lessons.

"I think previously, they didn't like hard thinking. They would like to be told what to do or the safety of it. But now their minds are being trained into being a hard thinker. Now they're expressing, 'give me a harder one!' "

Gervasoni,A. & Roche, A. (2018). *Insights about how to create flexible mathematics classrooms that enable all students to thrive.* In Hsieh, F.-J. (Ed.), (2018). *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*, Vol 2, Taipei, Taiwan: EARCOME., pp 388-397.

Insights and Implications

- **For some teachers, responding effectively to diverse student knowledge in a class was difficult.**
- **Ongoing professional learning that supports teachers to respond more inclusively and effectively to student diversity is important for school leaders to acknowledge and provide.**
- **For a large group of teachers, the greatest reported change in their teaching practice was their ability to cater for diversity. They discussed changes such as using more open tasks in mathematics lessons in order to respond flexibly to what each student needed to learn, and continually checking what children understood, to assist with planning lessons.**

Gervasoni,A. & Roche, A. (2018). *Insights about how to create flexible mathematics classrooms that enable all students to thrive.* In Hsieh, F.-J. (Ed.), (2018). *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*, Vol 2, Taipei, Taiwan: EARCOME., pp 388-397.

Conclusion

Teachers today play a key role in preparing all students to participate fully in a future global community in which rapid change will be one certainty.

We need inclusive classroom approaches and curricula that provide sufficient challenge and flexibility to enable all students to thrive mathematically, including those children who struggle.

Inquiry approaches provide the type of learning environment that best prepares children for learning and using mathematics in 2030.

Our Challenge as School Mathematics Leaders

How do we design mathematics classrooms that provide experience of the skills we predict are needed in 2030?

- **Creativity and innovation**
- **Critical thinking and problem solving**
- **Communication**
- **Collaboration**

Thank you for your Inspiration!