

# PROBLEMLØSNING - HVAD KAN DET?

UNDERVISNINGSDIFFERENTIERING I MATEMATIK

Søs Spahn  
1. maj 2019


KØBENHAVNS  
PROFESSIONS  
HØJSKOLE

# POLYA – HVEM OG HVAD?




- OPSLAG PÅ WIKIPEDIA:
- GEORGE PÓLYA (/ˈPOʊljə/; HUNGARIAN: PÓLYA GYÖRGY [ˈPOːljɒ ˈjø̞rɟʃ]) (DECEMBER 13, 1887 – SEPTEMBER 7, 1985) WAS A HUNGARIAN MATHEMATICIAN. HE WAS A PROFESSOR OF MATHEMATICS FROM 1914 TO 1940 AT ETH ZÜRICH AND FROM 1940 TO 1953 AT STANFORD UNIVERSITY. HE MADE FUNDAMENTAL CONTRIBUTIONS TO COMBINATORICS, NUMBER THEORY, NUMERICAL ANALYSIS AND PROBABILITY THEORY. HE IS ALSO NOTED FOR HIS WORK IN HEURISTICS AND MATHEMATICS EDUCATION.[2] HE HAS BEEN DESCRIBED AS ONE OF THE MARTIANS.[3]

"**The Martians**" was a term used to refer to a group of prominent Hungarian scientists (mostly, but not exclusively, [physicists](#) and [mathematicians](#)) who emigrated to the [United States](#) in the early half of the 20th century.

# PROBLEMLØSNING & PROCESORIENTERET MATEMATIKUNDERVISNING

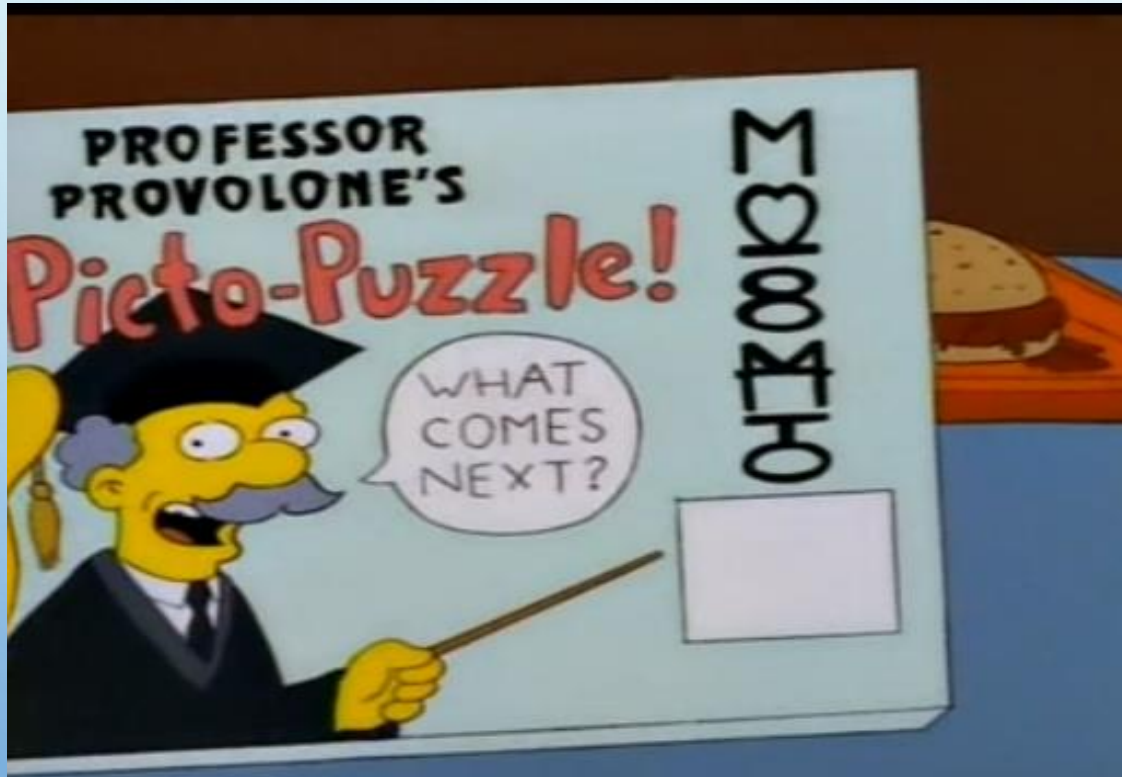


Assuming this pattern continued how many pool balls will be in the 10th pattern?

  Competition Winner 2016  NATIONAL FORUM 



# OPVARMNING!



## What Is Problem Solving?

*“Problem solving is solving non routine problems using skills that you have or gain in the solution of a problem which initially seemed unsolvable.”*

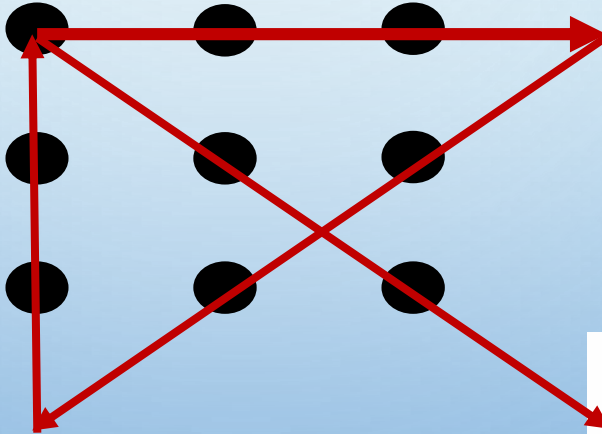
“A problem is only a problem if you don’t know how to go about solving it. A problem that can be solved comfortably by routine or familiar procedures is an exercise” (Schoenfeld, 1983)

DU ER ENGAGERET I PROBLEMLØSNING, NÅR DU PRØVER  
AT OPNÅ NOGET, OG DU IKKE HELT KAN FINDE DEN LIGE  
VEJ TIL LØSNINGEN



# FØRSTE UDFORDRING....

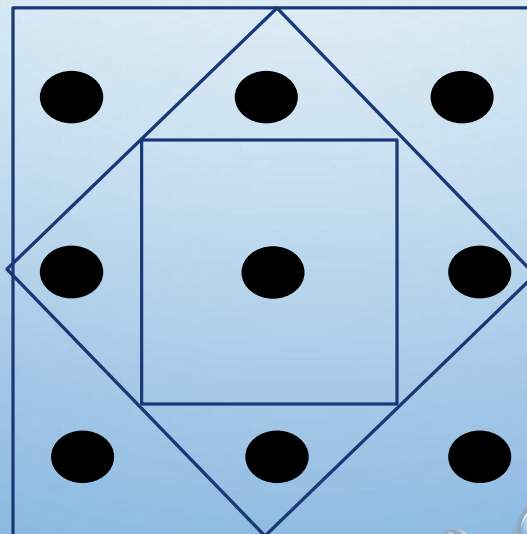
Ved hjælp af fire rette linjer skal du få disse ni punkter til at hænge sammen – uden at løfte blyanten fra papiret.





## UDVIDET UDFORDRING....

Inddel disse ni punkter ved at bruge tre kvadrater, så hvert punkt er i sit eget område (linjerne går ikke igennem/rører punkterne)





# HVORFOR ER PROBLEMLØSNING VIGTIG?

FOR ELEVER .....

- ER DET EN MÅDE AT LÆRE NY MATEMATIK
- ER DET EN MÅDE AT BRUGE SINE MATEMATISKE EVNER
- ER DET EN INDSIGT I MATEMATIKERES ARBEJDE
- OPBYGGENDE FOR SELVTILLIDEN OG PRODUCENT AF POSITIVE HOLDNINGER
- UDVIKLER DET UNDERSØGENDE OG RÆSONNERENDE EVNER
- FREMMER DET SAMARBEJDE OG KOMMUNIKATION
- ER DET UNDERHOLDENDE.

# HVORFOR ER PROBLEMLØSNING VIGTIGT?

- Life Skill



# WHY IS PROBLEM SOLVING IMPORTANT?

It gives students a taste for making knowledge rather than just receiving it (Neyland, 1995)

STUDENTS ACQUIRE WAYS OF THINKING AND **HABITS OF PERSISTENCE AND CURIOSITY** THAT SERVE THEM WELL OUTSIDE THE MATHEMATICS CLASSROOM (NCCA, 2005).

MATHEMATICAL KNOWLEDGE AND SKILLS HAVE LITTLE VALUE IF THEY CANNOT BE APPLIED TO NEW OR UNFAMILIAR SITUATIONS (UBUZ, 1994: 367)

# HVORFOR ER PROBLEMLØSNING VIGTIGT?

1128 SOLDATER SKAL TRANSPORTERES I BUSSE TIL DERES  
TRÆNINGSOMRÅDE. HVER BUS KAN TAGE 36 SOLDATER MED.  
HVOR MANGE BUSSE SKAL DER BRUGES FOR AT TRANSPORTERE  
ALLE SOLDATERNE?

(CARPENTER, LINDQUIST, MATTHEWS, & SILVER, 1983)

29% SVAREDE 31 REST 12

23% SVAREDE 32





# PROBLEMER MED PRØBLEMLØSNING

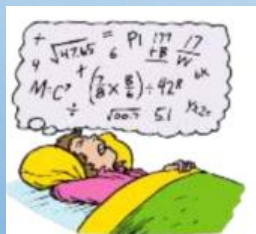
MANGLENDE TID OG TÅLMODIGHED I PROBLEMLØSNING.

“WHEN GIVEN A NOVEL TASK IN MATHS CLASS, STUDENTS ARE VERY LIKELY TO JUMP INTO THE PROBLEM WITH ONE STRATEGY, CONTINUE THE STRATEGY WITHOUT “LOOKING BACK” AND FINISH WITHOUT RE-EXAMINING THE SOLUTION...OFTEN THE RESULT CAN BE A MISUNDERSTOOD PROBLEM, OR AN INEFFECTIVE STRATEGY, AND/OR A SOLUTION THAT DOES NOT WORK”

(GOLDBERG & BUSH, 2003)

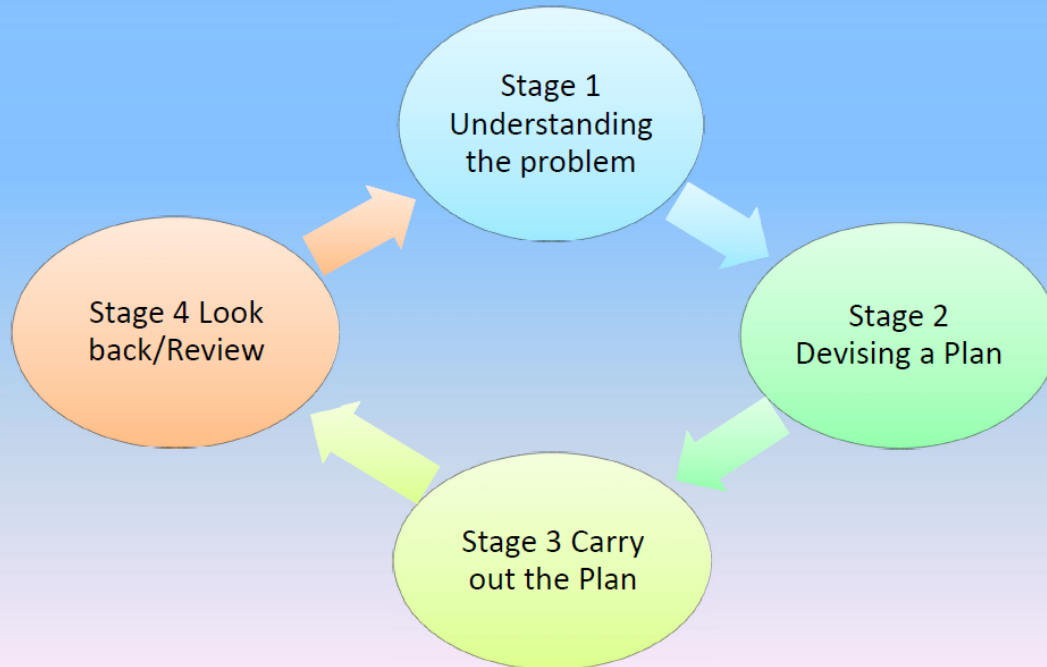
# TRADITIONEL MATEMATIK(OP)LÆRING ER OFTE INEFFEKTIV OG KEDELIG.

- WHEN STUDENTS TRY TO MEMORIZE HUNDREDS OF METHODS, THAT STUDENTS DO IN CLASSES THAT USE A PASSIVE APPROACH, THEY FIND IT EXTREMELY HARD TO USE THE METHODS IN ANY NEW SITUATION, OFTEN RESULTING IN FAILURE ON EXAMS, AS WELL AS IN LIFE.
- BOALER, J. THE ELEPHANT IN THE CLASSROOM, S. 36.



# HVAD ER PROBLEMLØSNING?

## Polya's 4 Stages of Problem Solving



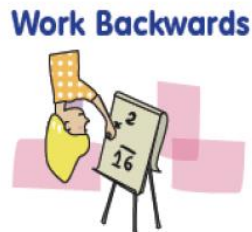
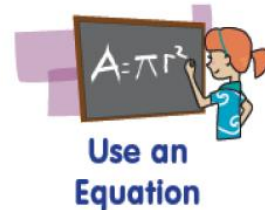
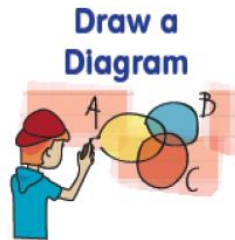
# 1. FORSTÅ PROBLEMET

- HVAD BLIVER DU BEDT OM AT FINDE ELLER VISE?
- KAN DU FORMULERER PROBLEMET MED DINE EGNE ORD?
- KAN DU FORESTILLE DIG ET BILLEDE ELLER ET DIAGRAM, DER KAN HJÆLPE DIG TIL AT FORSTÅ PROBLEMET?
- ER DER INFORMATION NOK TIL AT GØRE DET MULIGT FOR DIG AT FINDE EN LØSNING? HVIS JA, HVAD ER SÅ DEN VIGTIGSTE INFORMATION, DU HAR FÅET?



## 2. LÆG EN PLAN (RÆKKEFØLGE)

# Problem Solving Strategies



### 3. UDFØR PLANEN

- HOLD ØJE MED DINE FREMSKRIDT SÅ ALT GÅR EFTER PLANEN
- TJEK HVERT TRIN
- HUSK FEJLTRIN ELLER SÆRLIGE OVERVEJELSER IFT. FREMTIDIG GENNEMGANG
- PRØV EN NY PLAN, HVIS DENNE IKKE LYKKES



## 4. REFLEKTER – SE TILBAGE

- ER DER EN METODE, DU KAN BRUGE, SOM TILLADER DIG AT VERIFICERE DIT SVAR?
- GIVER DIT SVAR MENING?
- KAN DU SÆTTE DIT SVAR IND I SPØRGSMÅLETS KONTEKST?
- REFLEKTER OG SE TILBAGE PÅ DIN PROCES – HVAD VIRKEDE? HVAD VIRKEDE IKKE?



# Problem Solving Model

## Read It!

### UNDERSTAND THE PROBLEM

- Read the problem 2, maybe 3 times. Highlight or underline important information.
- Talk it! – talk about the problem to understand it better.



## Think It!

### MAKE A PLAN

- What strategy will you use and why?
- Talk it! – discuss strategies with a partner.
- What manipulatives will you use?



## Solve It!

### CARRY OUT THE PLAN

- Apply your strategy.
- You may need to revise and try a different strategy.....
- Show your work (thinking).
- Ask yourself...
  - Is your answer reasonable?
  - Does it make sense?



## Explain It!

### COMMUNICATE THE SOLUTION

- Answer the question!
- Tell, show, write, ... how the answer was reached. Consider extensions.
- First I... I noticed that... Then I... I thought... Finally...





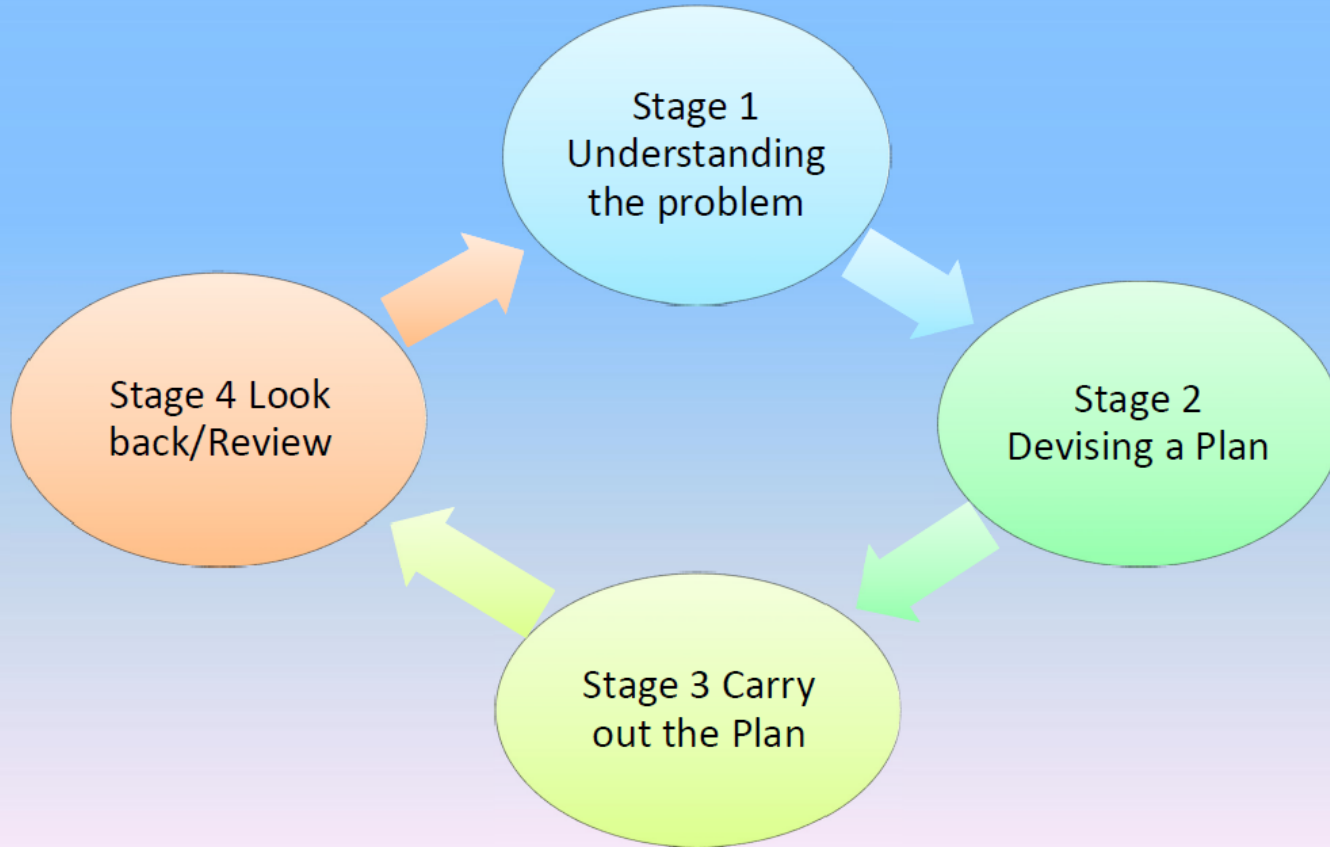


## PROBLEM 1



- JOHN HAR 18 TI-KRONER I SIN PUNG OG KARL HAR 22 FEM-KRONER I SIN PUNG
- DE BESLUTTER, AT DE HVER DAG VIL TAGE EN MØNT FRA DERES PUNGE OG LÆGGE I EN SPAREBØSSE, INDTIL EN AF DEM IKKE HAR FLERE PENGE I SIN PUNG.
- HVORNÅR HAR KARL FLERE PENGE END JOHN I SIN PUNG?

# Stages of Problem Solving

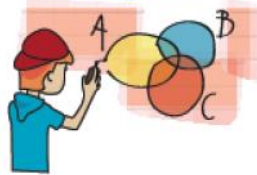


# Problem Solving Strategies



**Trial and Improvement**

**Draw a Diagram**



**Look for a Pattern**

**Act It Out**



**Draw a Table**

**Simplify the Problem**



**Use an Equation**

**Work Backwards**



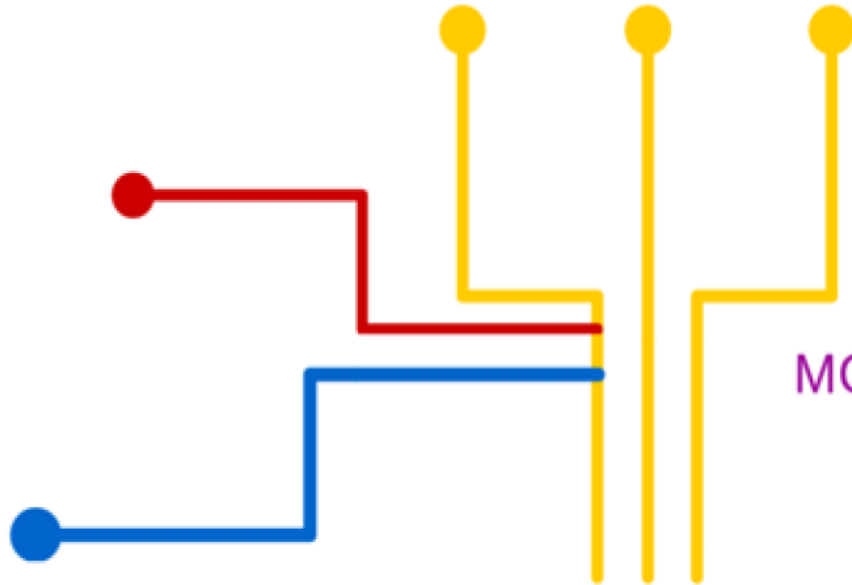
**Eliminate Possibilities**

# HVORDAN KAN DET LØSES?

- BUD PÅ LØSNINGSMULIGHEDER.....



In problem solving there may always be...



MORE THAN ONE  
WAY.....

## MAKING A TABLE

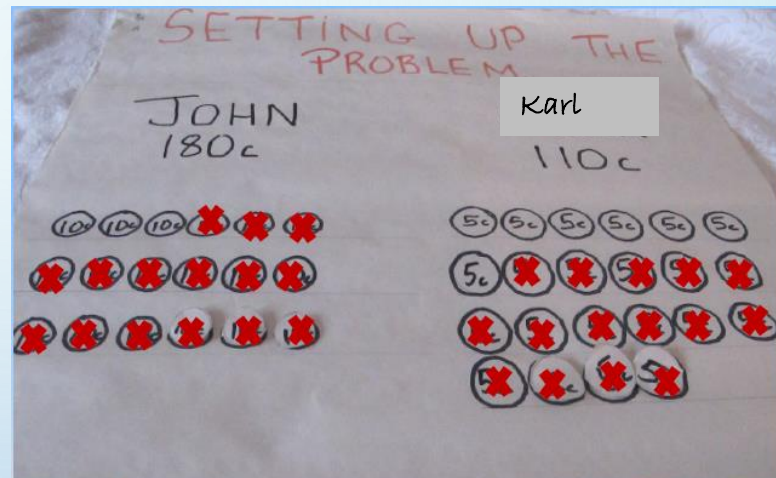
JOHN

No OF DAYS	1	2	3	...	14	15	16	17	18
AMOUNT LEFT	170	160	150	...	40	30	20	10	0

Karl

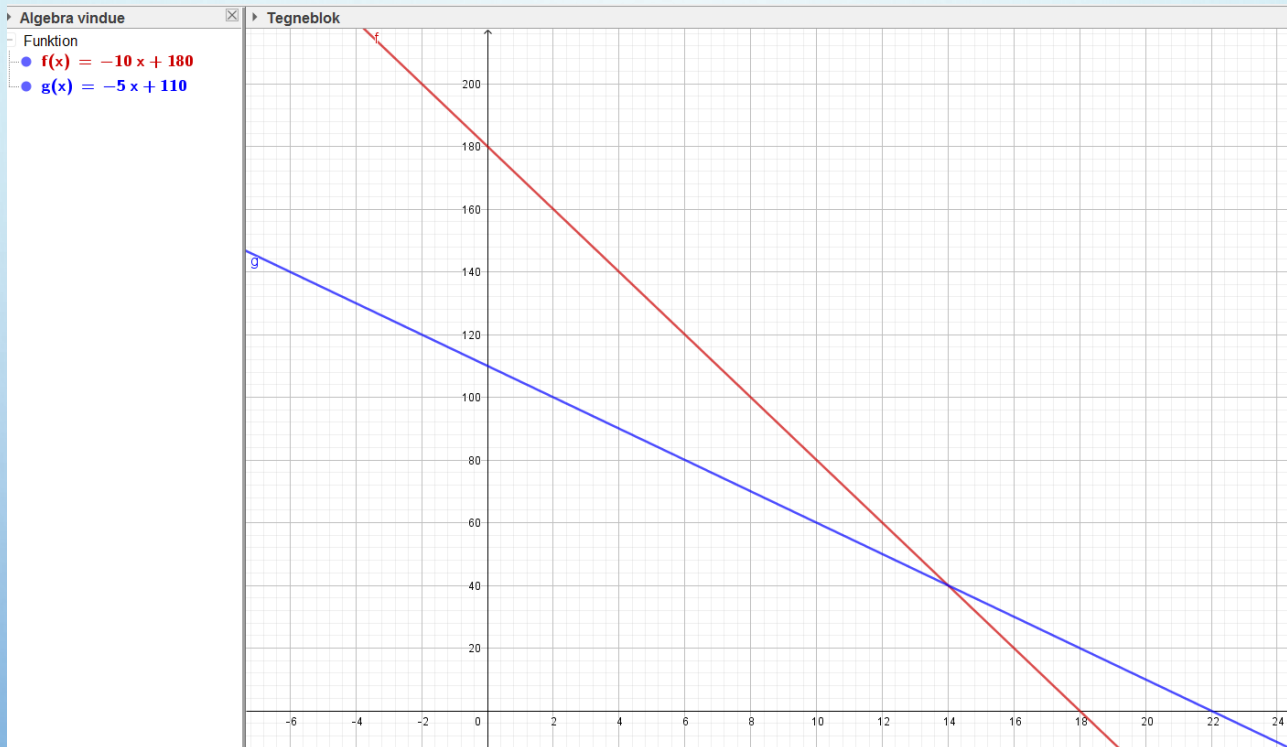
No OF DAYS	1	2	3	...	14	15	16	17	18
AMOUNT LEFT	105	100	95	...	40	35	30	25	20

ANSWER 15TH DAY



MANIPULATING  
ACTUAL OBJECTS.

ANSWER 15TH DAY  
(BY CROSSING THEM OFF).



5/1/2019

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











## PROBLEM 2

- HVILKET TAL MANGLER?

				9
				5
				14
				8
9	8	8	?	



# LØSNING A

				9
				5
				14
				8
9	8	8	?	

3 gule og 1 rød  
Gul kunne være 1 og så er rød  
2

3 blå og 1 rød  
Hvis rød er 2, så er blå:  
 $\frac{12}{3} = 4$

2 blå + 1 gul + 1 rød  
 $2(4) + 1(1) + 1(2) =$   
11

# LØSNING B

ET SÆT AF LIGNINGER FX:

				9
				5
				14
				8
9	8	8	?	

$$\left\{ \begin{array}{l} 2x + y + z = 9 \\ 3y + x = 5 \\ x + 3z = 14 \end{array} \right.$$

# LØSNING C

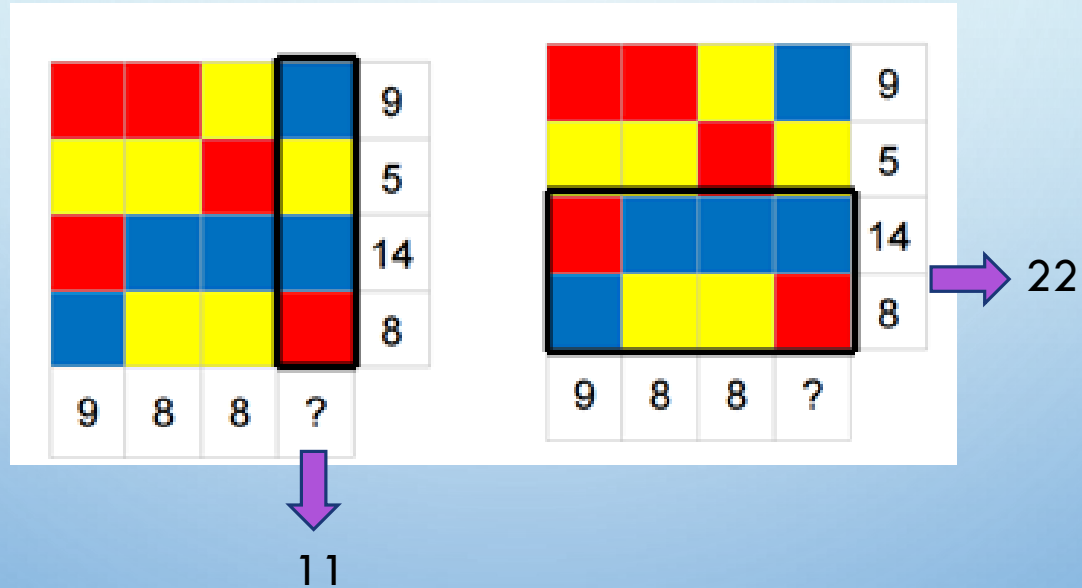
				9
				5
				14
				8
9	8	8	?	

$$\left. \begin{array}{l} 9 \\ 5 \\ 14 \\ 8 \end{array} \right\} = 36$$

$$? = 36 - 25$$

$$? = 11$$

# LØSNING D





# KVÆLER VI KREATIVITET I SKOLENS MATEMATIKUNDERVISNING?

DET FOREGÅENDE PROBLEM BLEV GIVET TIL TO GRUPPER  
AF ISRAELSK ELEV/STUDERENDE FRA FORSKELLIGE TRIN  
(UZIEL & AMIT, 2016)....

- 24 ELEVER I ALDEREN 11-12 LØSTE PROBLEMET. 1 AF  
DEM ANVENDTE ALGEBRA.
- 31 STUDERENDE I ALDEREN 17-18 LØSTE PROBLEMET. 25  
AF DEM ANVENDTE ALGEBRA.

# NU ER DET JERES TUR

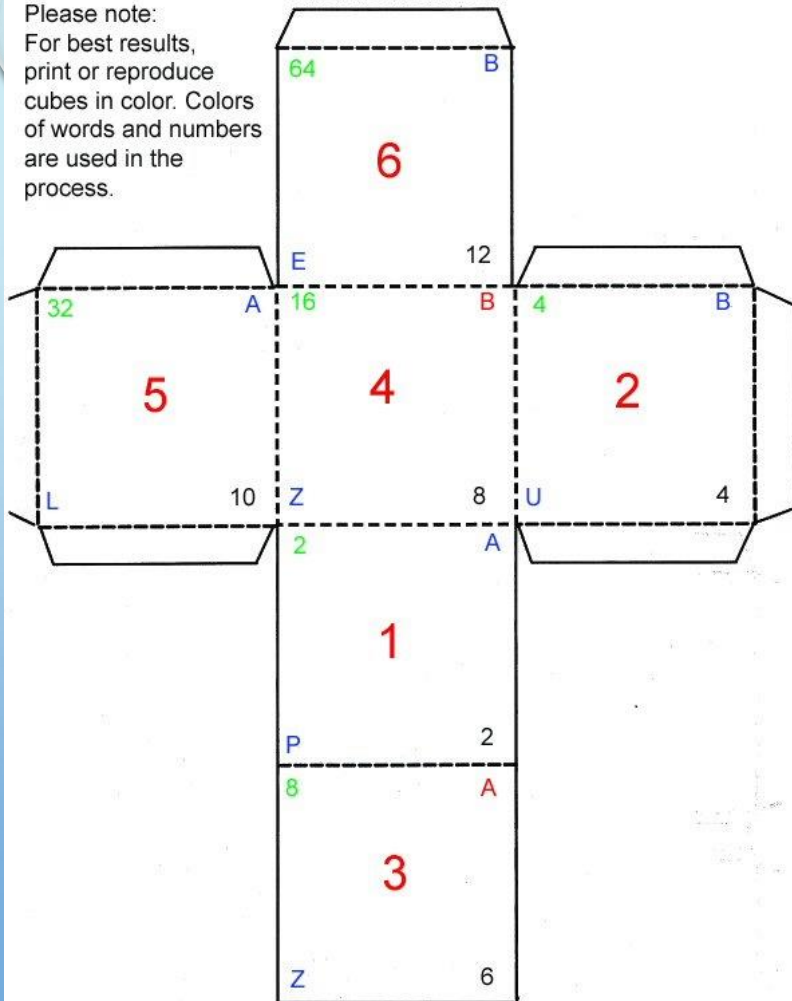
- HVORDAN VIL I TILRETTELÆGGE UNDERVISNINGEN I JERES KLASSE, HVIS I SKAL TILGODESE ALLE ELEVER?
- HVILKE PROBLEMER KAN I STILLE, HVIS UNDERVISNINGSDIFFERENTIERINGEN SKAL LYKKES?
- TAG EVT. UDGANGSPUNKT I OPLÆGGENE FRA 'AT OPSTILLE PROBLEMER'

# LITTERATUR OG LINKS

- JO BOALER – THE ELEPHANT IN THE CLASSROOM
- POLYA – HOW TO SOLVE IT
- DAN FINKEL – TEDTALK: [HTTPS://WWW.YOUTUBE.COM/WATCH?V=YTVNEQUA5-C](https://www.youtube.com/watch?v=YTVNEQUA5-C)
- DAN FINKEL – MATHFORLOVE (BILLEDER): [HTTPS://MATHFORLOVE.COM/LESSON/PRIME-CLIMB-COLOR-CHART/](https://mathforlove.com/lesson/prime-climb-color-chart/)
- ERASMUS+ PROJEKT: [#PROBLEM SOLVING](#)
- **LÅN AF MATERIALER: MITCFU.DK – BÅDE KONKRETE MATERIALER, BØGER OG TV/FILM – MED PÆDAGOGISKE VEJLEDNINGER OG KAPITELMÆRKNING (OGSÅ TIL MATEMATIK)**

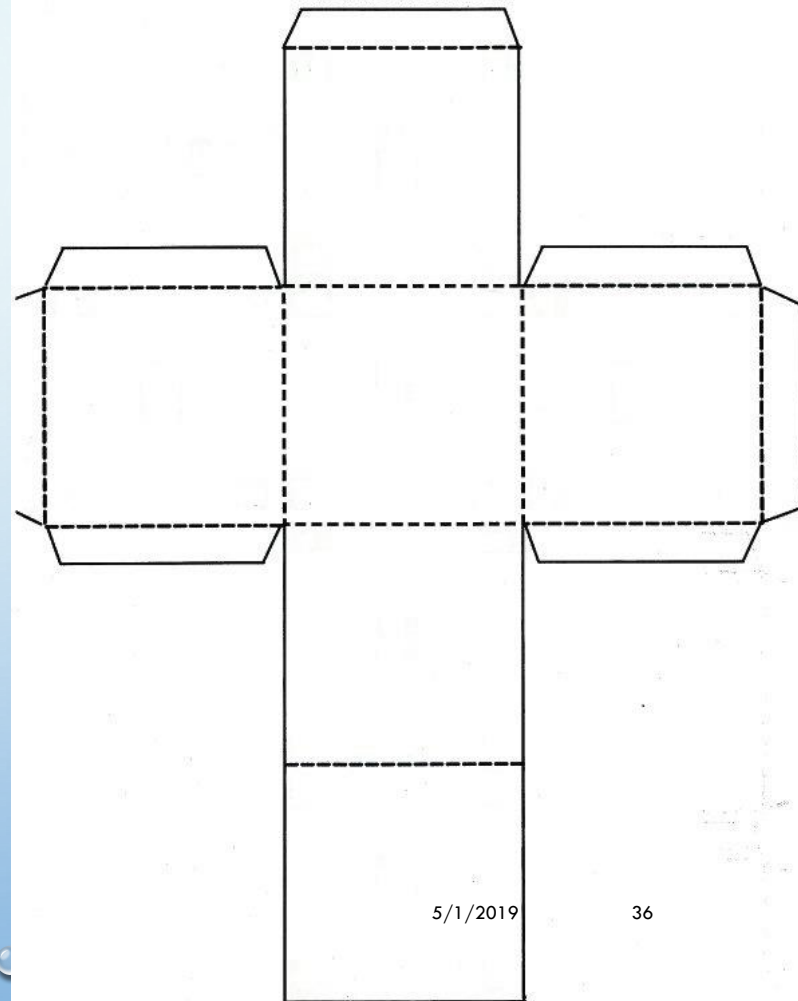
Cut on solid lines · Fold on dashed lines

Please note:  
For best results,  
print or reproduce  
cubes in color. Colors  
of words and numbers  
are used in the  
process.



# TERNINGER

Cut on solid lines · Fold on dashed lines





TAK FOR I DAG!

