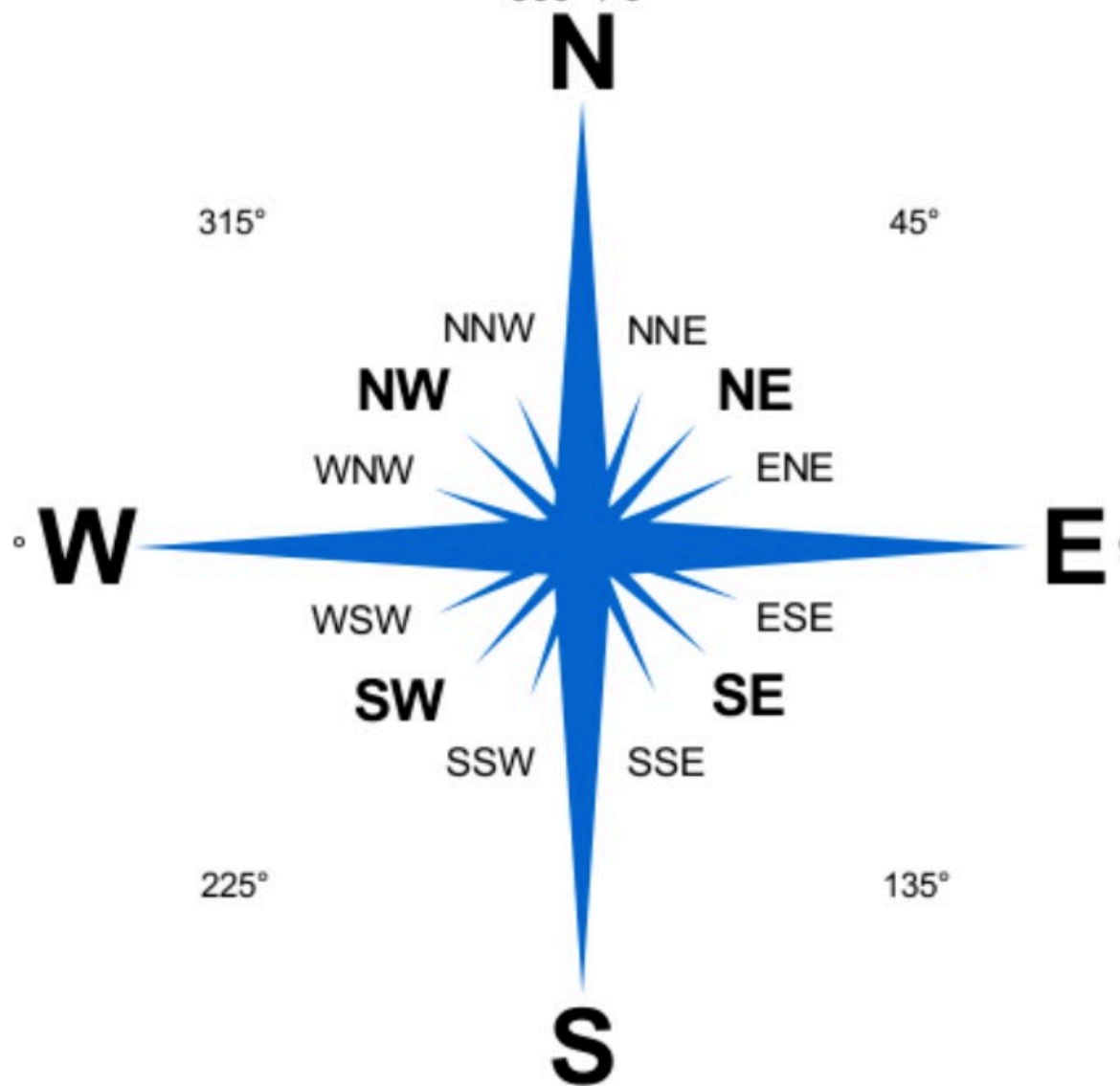


Which Way to the North Pole?

The north magnetic pole is moving eastward at an accelerating pace. Is doomsday nigh?

**Which Way With
Math Education?**

4 CARDINAL DIRECTIONS

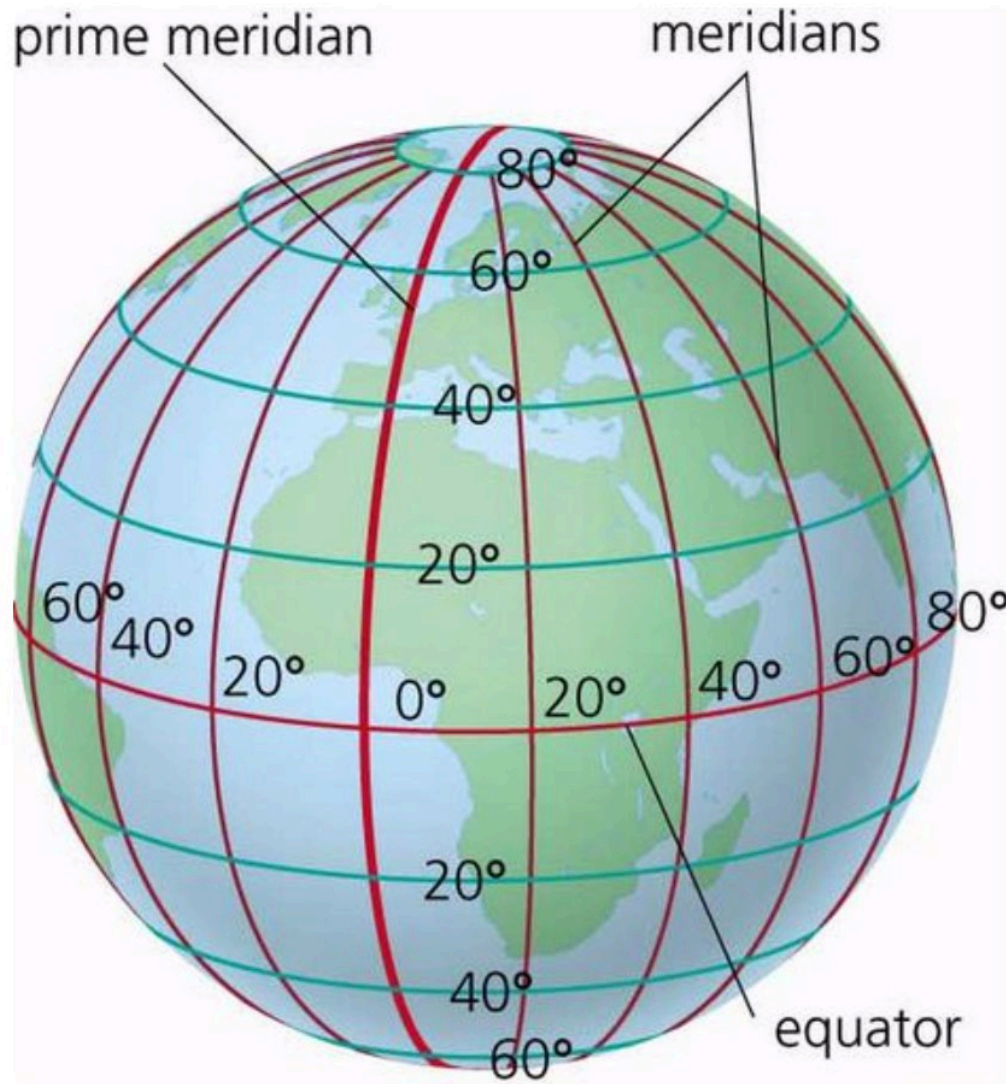


NORTH POLE

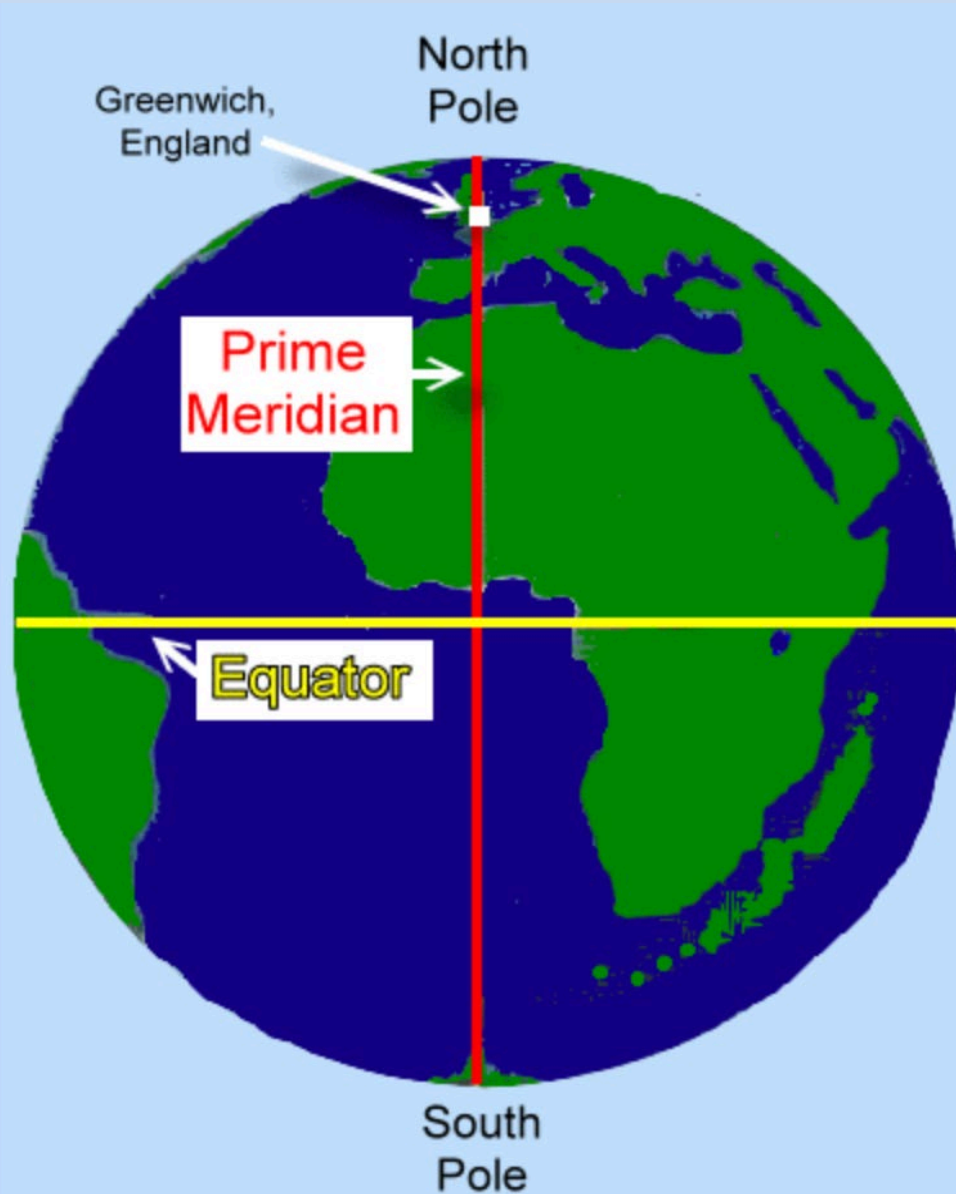
- WE ALL KNOW THE NORTH POLE
- How about the

WESTPOLE?

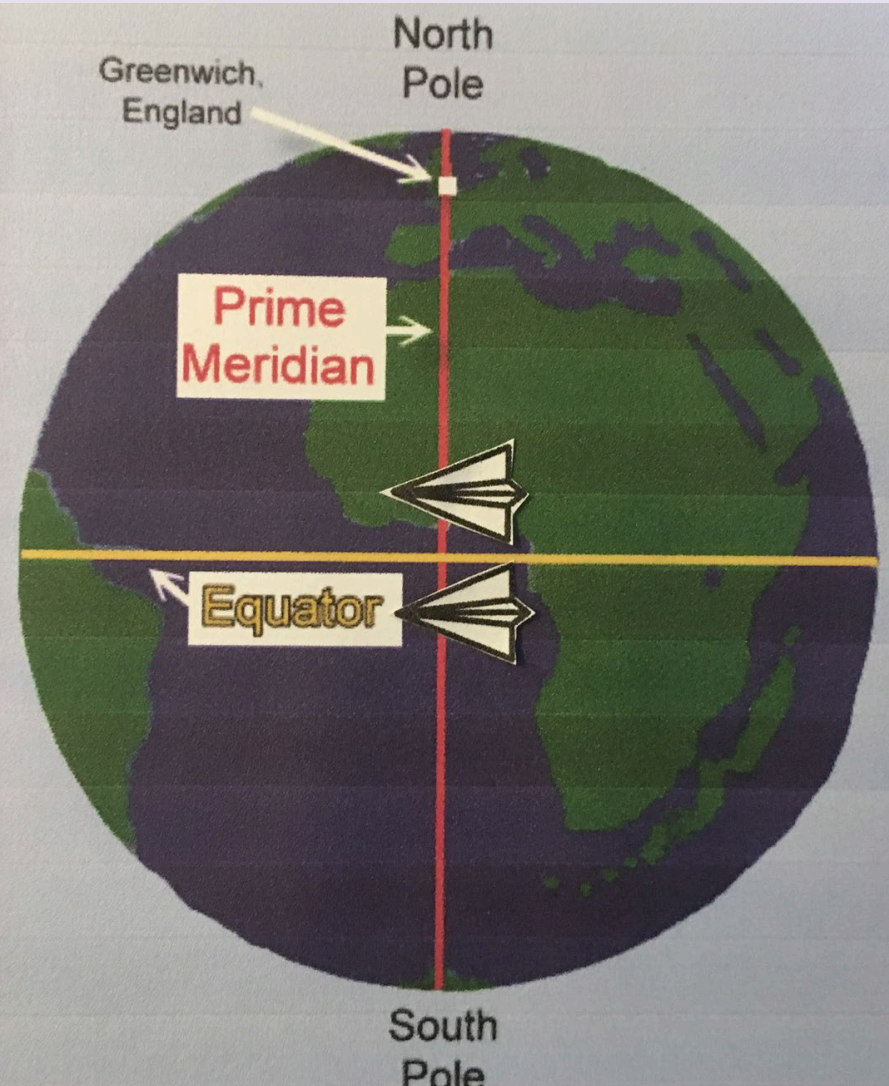
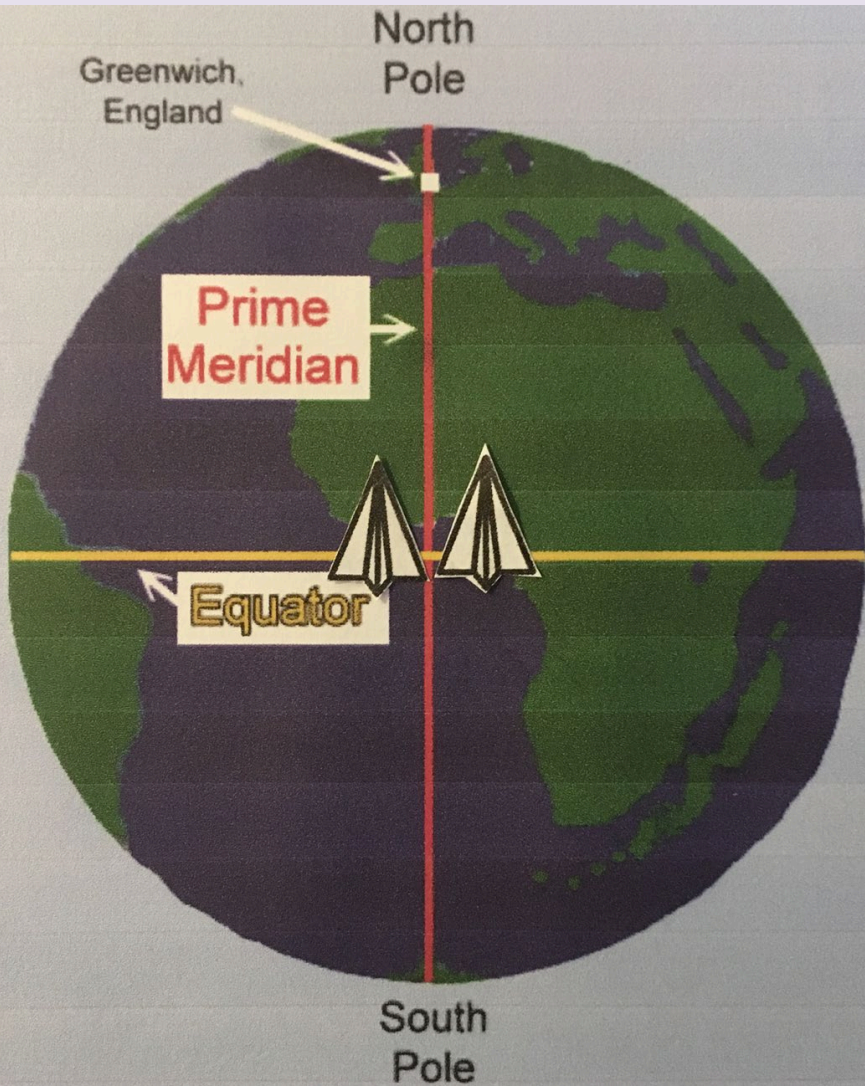
OUR COORDINATE SYSTEM



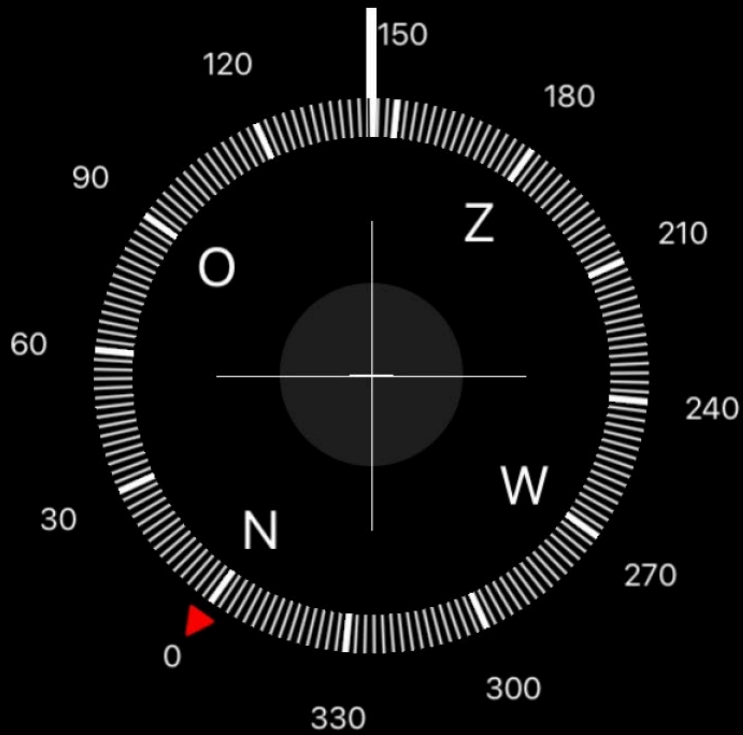
THE BASIS



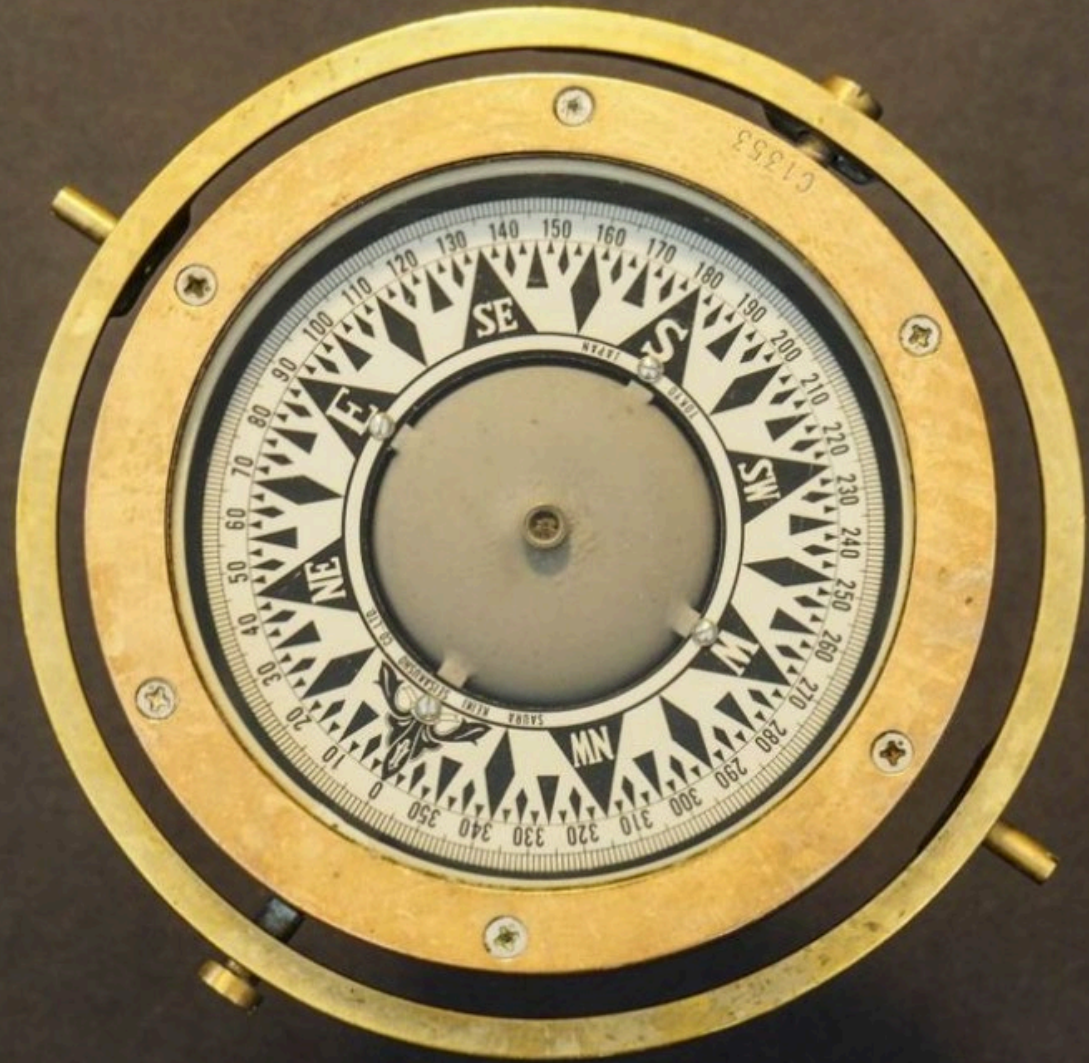
FLY THE DIFFERENCE



NORTHPOLES ?

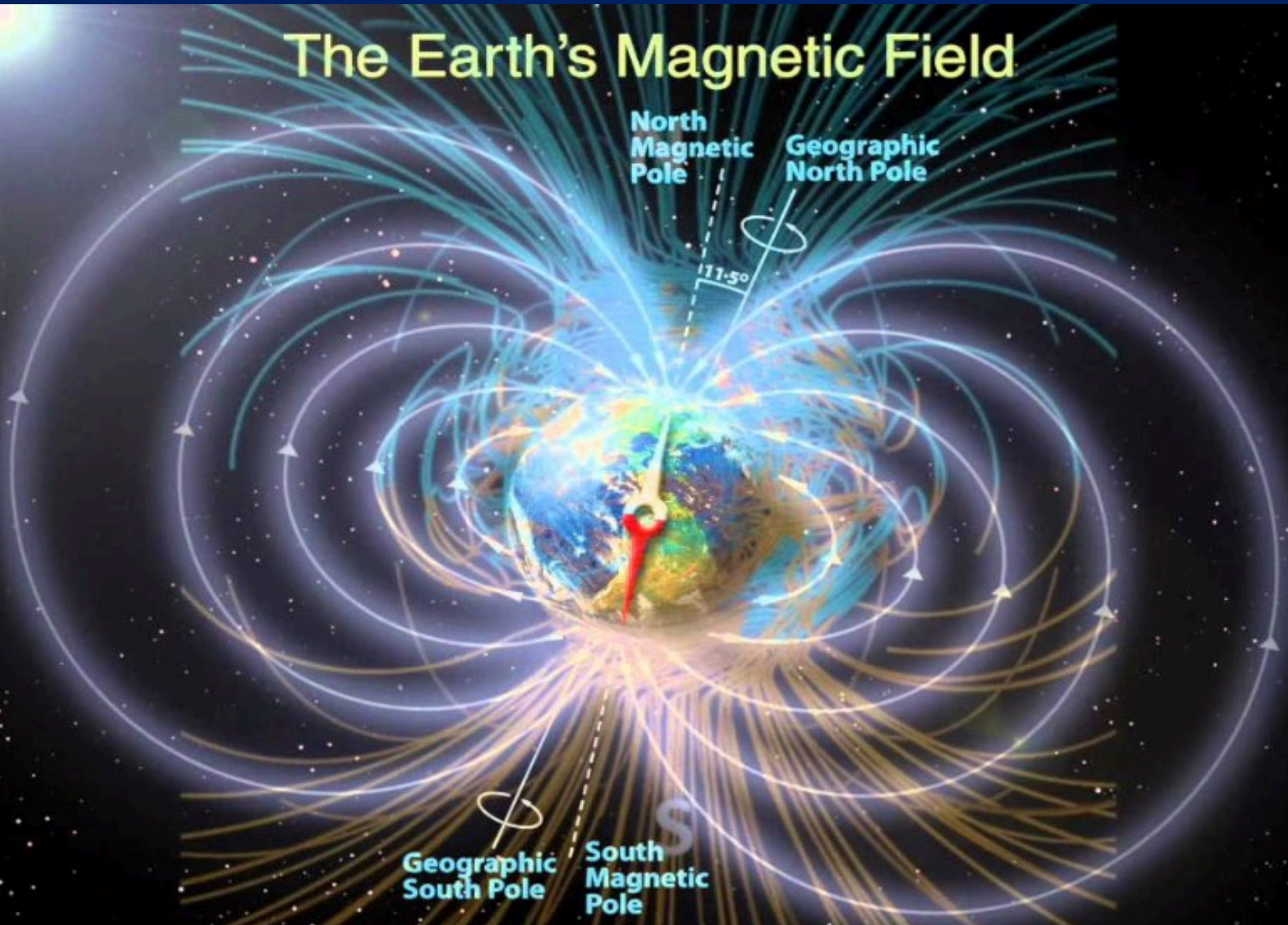


144° ZO

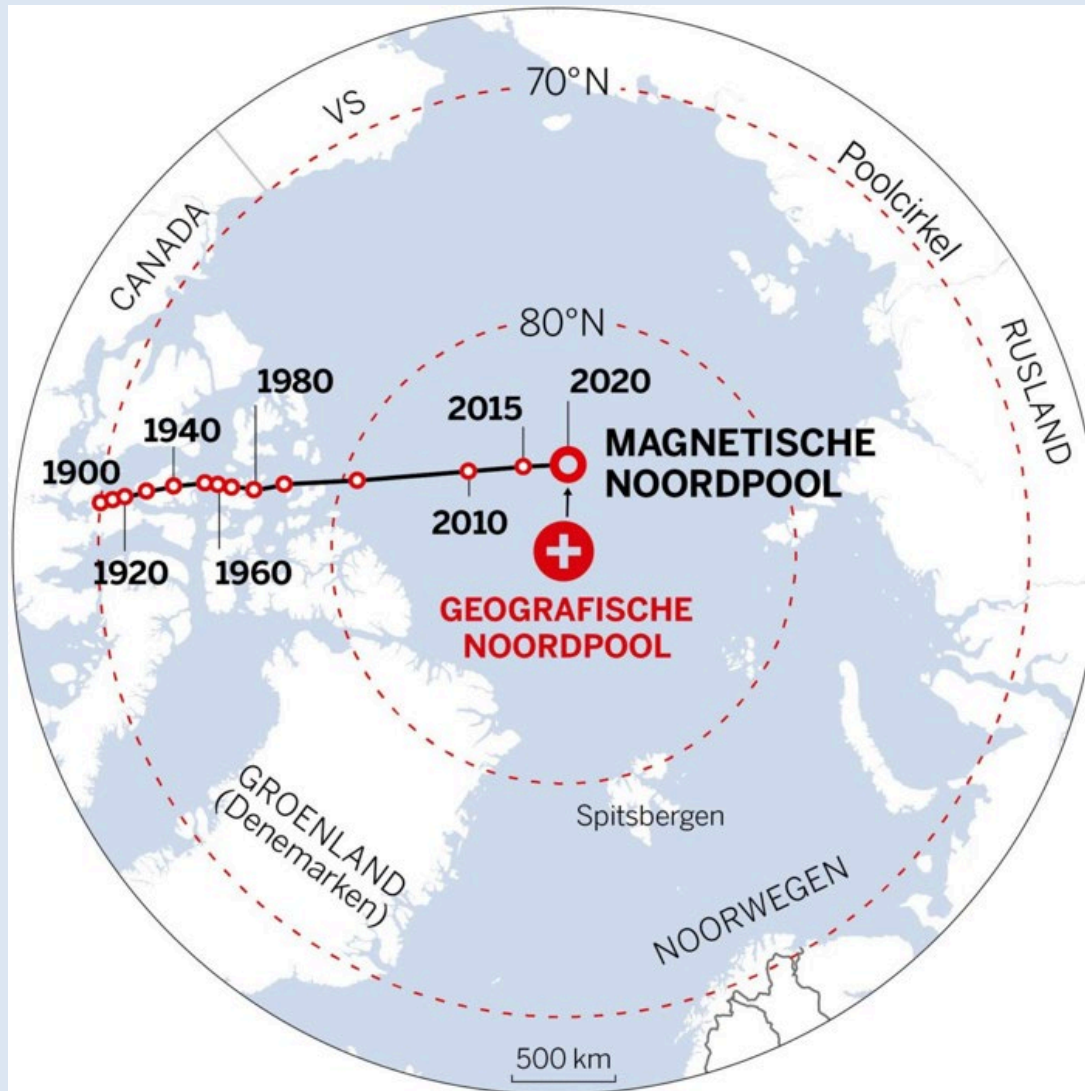


MAGNETIC POLE & NORTH POLE

The Earth's Magnetic Field



NORTH POLE: WALKING TO PUTIN?

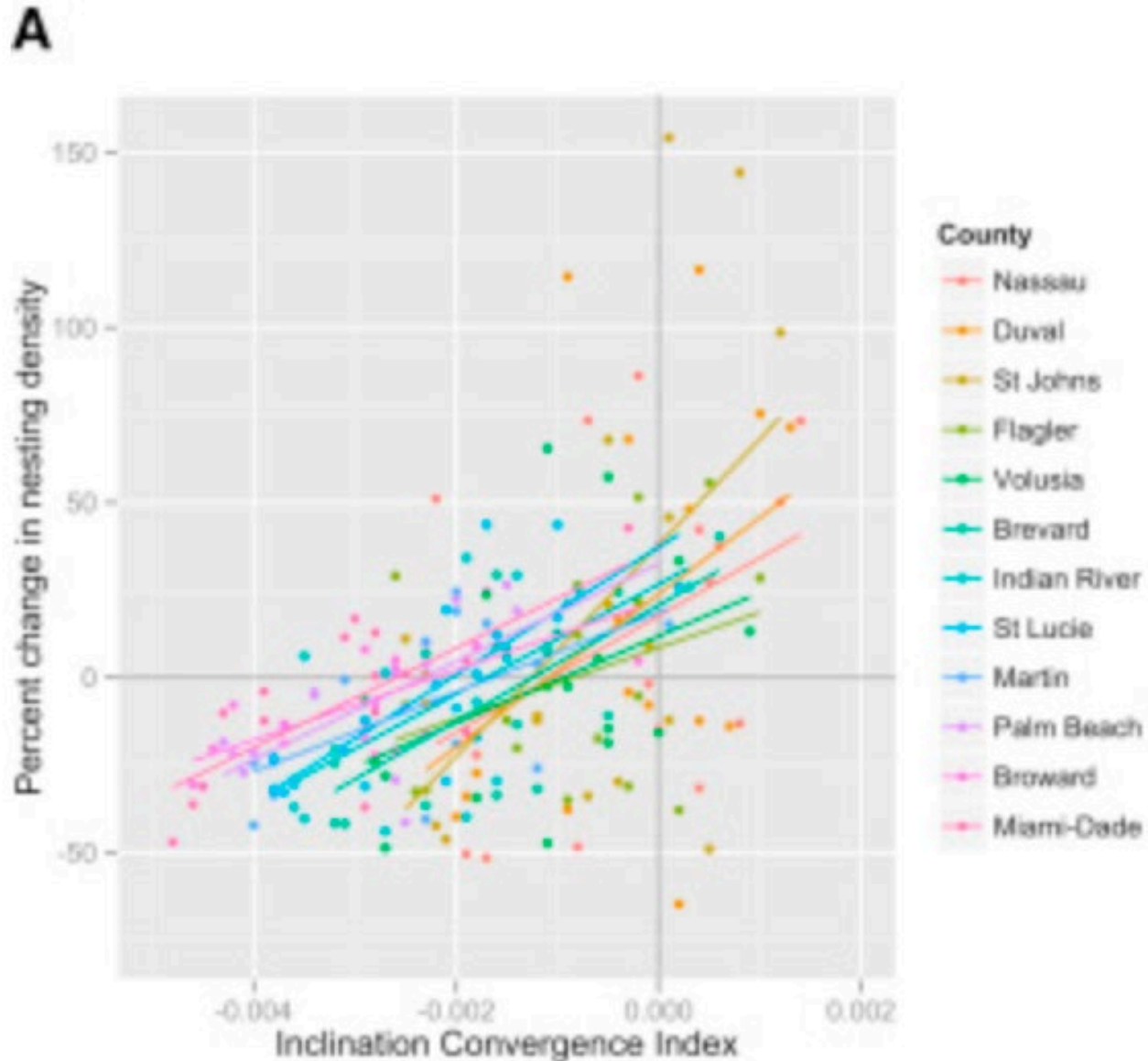


55 km per year

HOW ABOUT THE SEA TURTLE?



THE SEA TURTLE FINDS ITS WAY MAGNETICALLY



MATH: WHAT IS THE DIRECTION?

TIME Magazine 2013:

- “Young people (teens & twens) are known for constantly holding up cameras, taking selfies, and posting them online”
- ‘They are narcissistic, overconfident, entitled, and lazy”
- NOW, imagine being used to this technology your whole life.... *And having to sit through Algebra*

“How to teach mathematics as to be useful?”

(Freudenthal 1968)

- To TEACH
- To LEARN

- What is Mathematics?
- Why do we teach it?

- Teaching & Learning by Doing, based on Curiosity
- (de Lange, 2005)

Dewey & The U of Chicago Laboratory School

- 1895
- ‘The primary school serves the purpose to keep the theoretical work in touch with the demands of practice and make an experimental station for testing and developing of methods’ (Dewey, 1896)
- “No student seem to have a book, but they all seem to have such a good time” (Runyan, 1900)

Experimentation and Curiosity

Keypoints:

- Experimentation & Curiosity
- “Dewey is a moderate: a mix of the ‘transmission’ method (William Torrey Harris) and developmental approach, capitalizing on the instincts of children” (Kiebard 2004)

IMPACT of DEWEY

- Minimal
- Substantial:
 - Experimentation, learning from failure and constant inquiry (Sarason, 1982)
 - Relating curriculum to children's life and experience, integrating curriculum, teaching critical thinking and problem solving, stimulating creative thinking.(Tanner, 1997)

WESLEY

- The video



TOY/INSTRUMENT

CURIOSITY

CHILD

ADULT



- EXPLORE

- DO &
THINK

- REFLECT



WESLEY: INQUIRY BASED

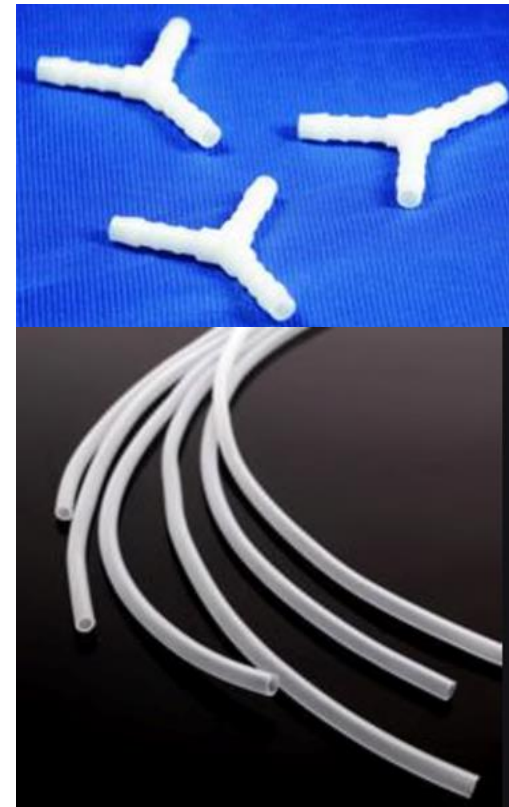
- **EXPLORATION**
- **THINKING & DOING**
- **REFLECTION**

- **CHALLENGE FOR TEACHER: INTERACTION**
- **POSING RIGHT QUESTIONS**
- **CHARACTERISTICS: EXPLORATION, CHALLENGE**

WHERE IS THE MATH ?

RATIO & PROPORTION, MEASUREMENT

- REASONING & THINKING
- Developing further mathematical ideas:



aandrijving



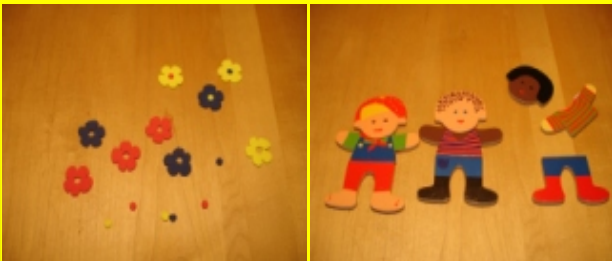
meten



snelheid, steilte



combinaties



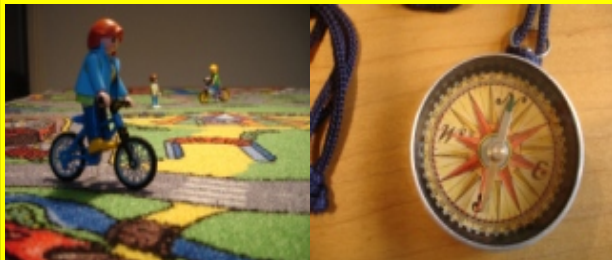
optische fenomenen



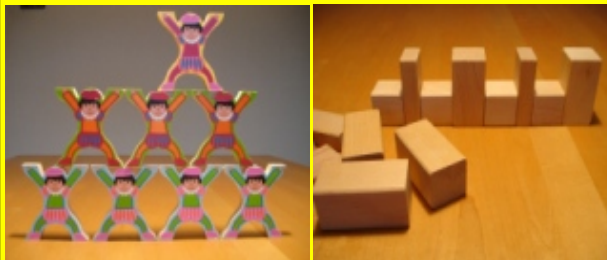
sorteren, classificeren



navigeren



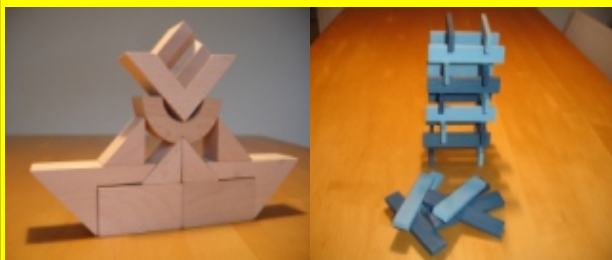
series, patronen



schaal, proportie



constructie



oriëntatie



tellen, nummers



PROBLEM SOLVING

- Math is one of the worlds greatest systems of
PROBLEM SOLVING

- Conrad Wolfram, 2013

PROBLEM SOLVING

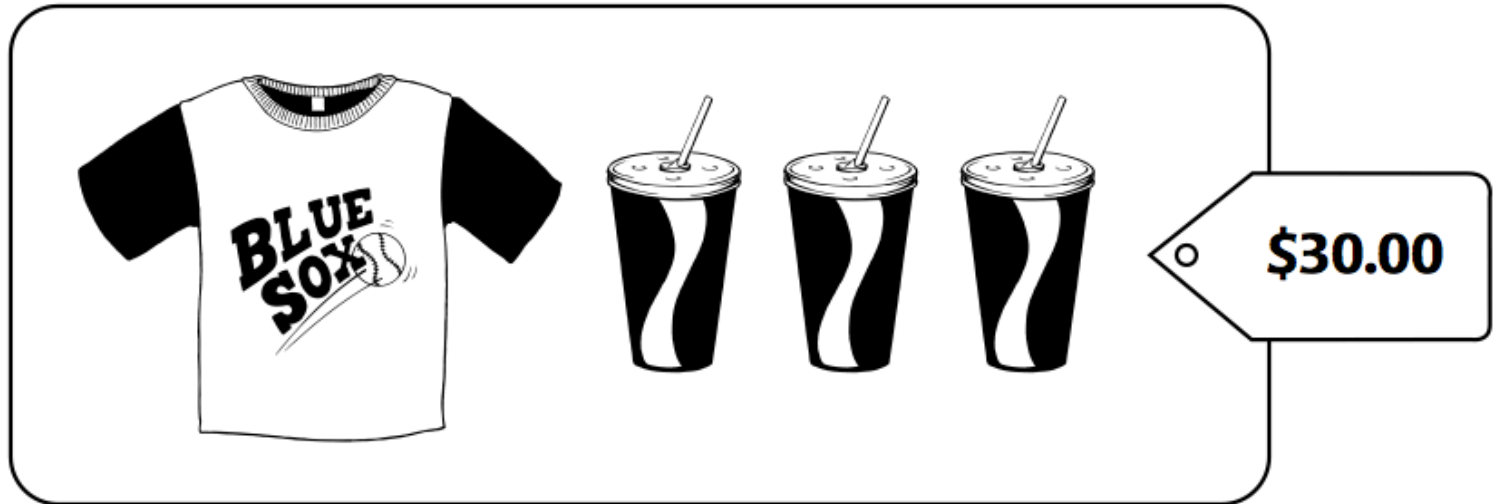
"Problem solving means engaging in a task for ***which the solution method is not known*** in advance.

In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings.

Solving problems is not only a goal of learning mathematics, but also a major means of doing so."
(NCTM, 2000, p. 52)

2nd PURPOSE

- PROBLEM SOLVING TO
DEVELOP
MATHEMATICAL
CONCEPTS



1. How much does a T-shirt cost?

How much is a drink?

Explain how you got your answers.

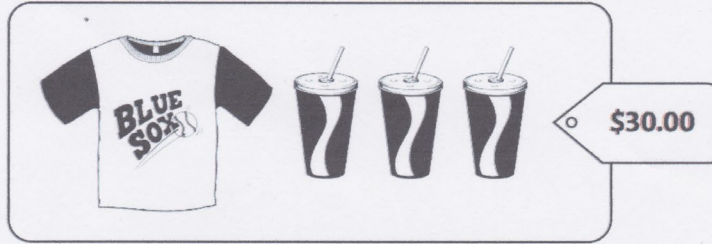


1. How much does a T-shirt cost?

How much is a drink?

Explain how you got your answers.

one t-shirt costs \$ 18.00
because 1 t-shirt and 1 soda
are \$ 22.00, this leaves
2 sodas in the lower picture
and \$ 8.00 so 1 soda is \$ 4.00
and $22 - 4 = 18$ so 1 t-shirts
costs \$ 18.00



1. How much does a T-shirt cost?

How much is a drink?

Explain how you got your answers.



2 shirts and 2 cups then:

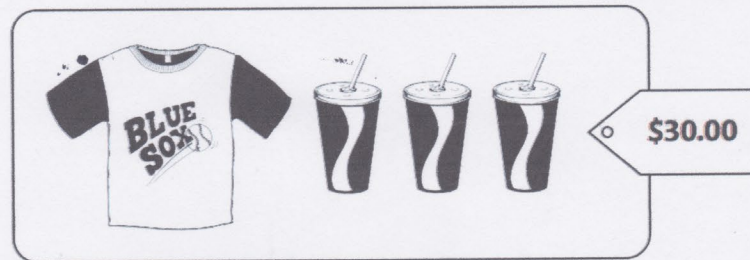
1 shirt and 3 cups then:

0 shirt and 4 cups -

almost ready

Price was 44.- less 1 t-shirt
30.- less 1 t-shirt

$$\text{left } 16 \div 4 = 4$$



1. How much does a T-shirt cost?

How much is a drink?

Explain how you got your answers.

cup $30 - 22 = 8$ for 2

so
 $44 - 8 = 36$ for 2 t-shirts

18 for 1 t-shirts

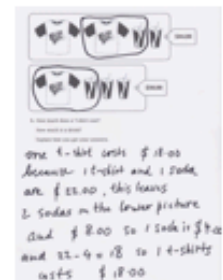
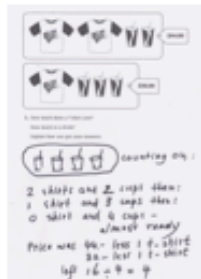
1 cup \$ 4.00

1 shirt \$ 18.00



1. How much does a T-shirt cost?
 How much is a drink?
 Explain how you got your answers.

CONCEPTUAL MATHEMATIZATION



INTERACTION SCHEMATIZATION FORMALIZATION

$$2x + 2y = 44$$

$$x + 3y = 30$$

CONCEPTS

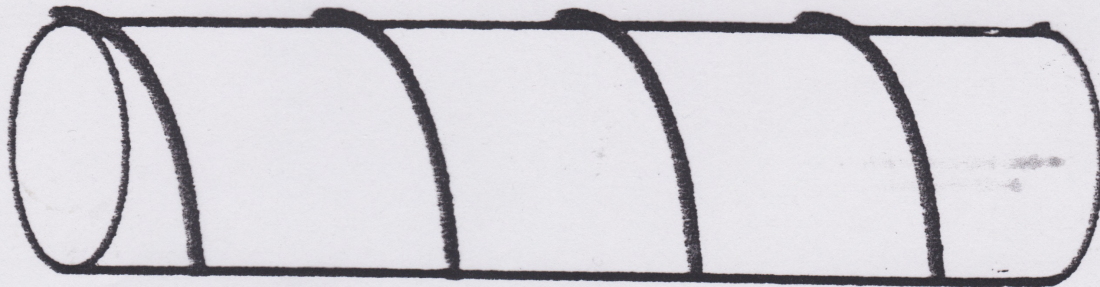
VARIABLE EQUATION

APPLIED MATHEMATIZATION

REINFORCING CONCEPTS

The Problem

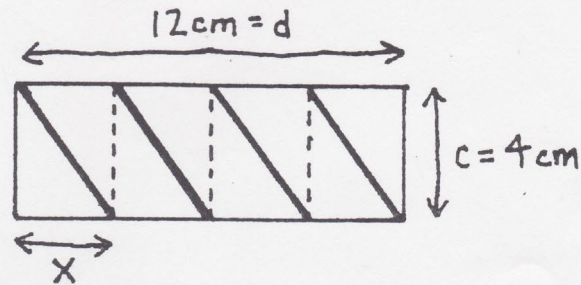
A string is wound symmetrically around a circular rod. The string goes exactly four times around the rod. The circumference of the rod is 4 cm. and its length is 12 cm.



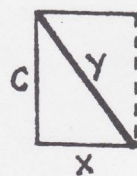
Find the length of the string.
Show all your work.

The Solution

Imagine that you unwrap the cylinder and flatten it.



$$x = \frac{d}{4} = \frac{12\text{ cm}}{4} = 3\text{ cm}$$



$$y^2 = c^2 + x^2$$

$$y^2 = 16 + 9 = 25\text{ cm}^2$$

$$y = 5\text{ cm}$$

$n =$ number of times string is wrapped around $= 4$

$$\text{Length of string} = y \cdot n = 5\text{ cm} \cdot 4$$

$$= \underline{\underline{20\text{ cm}}}$$

New York Times

Mathematics is
Thinking

INQUIRY BASED LEARNING

- And
- Constructivist
- Discovery
- Problem Based
- Experiential
- THEY ARE ALL PART OF
- **MINIMAL GUIDANCE INSTRUCTION**
- (Kirschner, Sweller, Clark 2006)

FAILURES

- Why **Minimal Guidance During Instruction** Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem Based, Experiential, and Inquiry-Based Teaching
- Why **Direct Instruction Does Not Work** for Problem Solving: An Analysis of the Failure to Understand the Mathematics, Teaching and Learning and Problem Solving

THE PROBLEMS WITH KIRSCHNER/SWELLER

- Failure to describe:
- What is MINIMAL here? (Minimally can be Maximum as well)
- They do not define: Mathematics, Learning, Teaching, Problem Solving
- They do not see differences between 5 methods
- They adapt **one** brain theory (Sweller's)
- Sweller's research is based on wrong ideas about mathematics and poor research
- (See: Treffers 2019)

SIMILAR PROBLEMS AS WITH HATTIE

- NO DISCRIMINATION OF ALL TEACHING STYLES
- NO CAREFUL META-ANALYSIS OF MANY DIFFERENT META ANALYSES
- CONCLUSIONS: BAKED AIR

BEST TEACHING IF TEACHER **BELIEVES** IN HIS CHOSEN METHOD (ROMBERG, SHAEFFER 2002)

PROBLEM BASED EDUCATION: EXTRA COMPETENCIES ON Problem solving

INDICATION of PROOF: PISA

PROBLEM SOLVING & LITERACY

- The ability to pose, formulate, solve and interpret problems using mathematics within a variety of situations.
- Identify and understand the role that mathematics plays in the real world
- (OECD/PISA 2009)

COMPETENCIES FOR THE FUTURE

CREATIVE & CRITICAL THINKING
& REASONING

PROBLEM SOLVING

LOGICAL REASONING

Future competencies

Last 15 years:

SEEING THESE

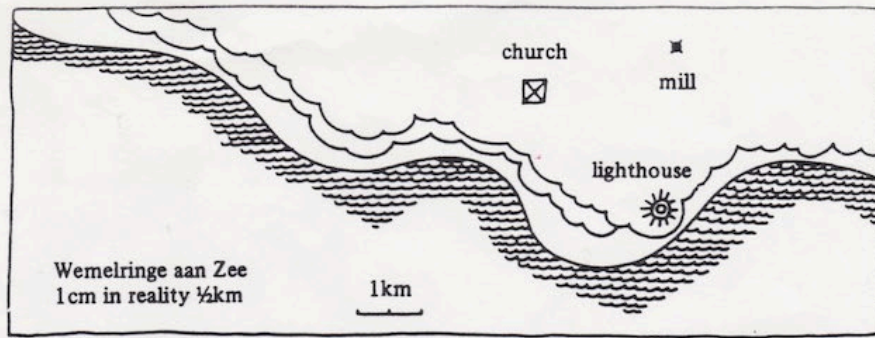
COMPETENCIES

DEVELOP OVER TIME



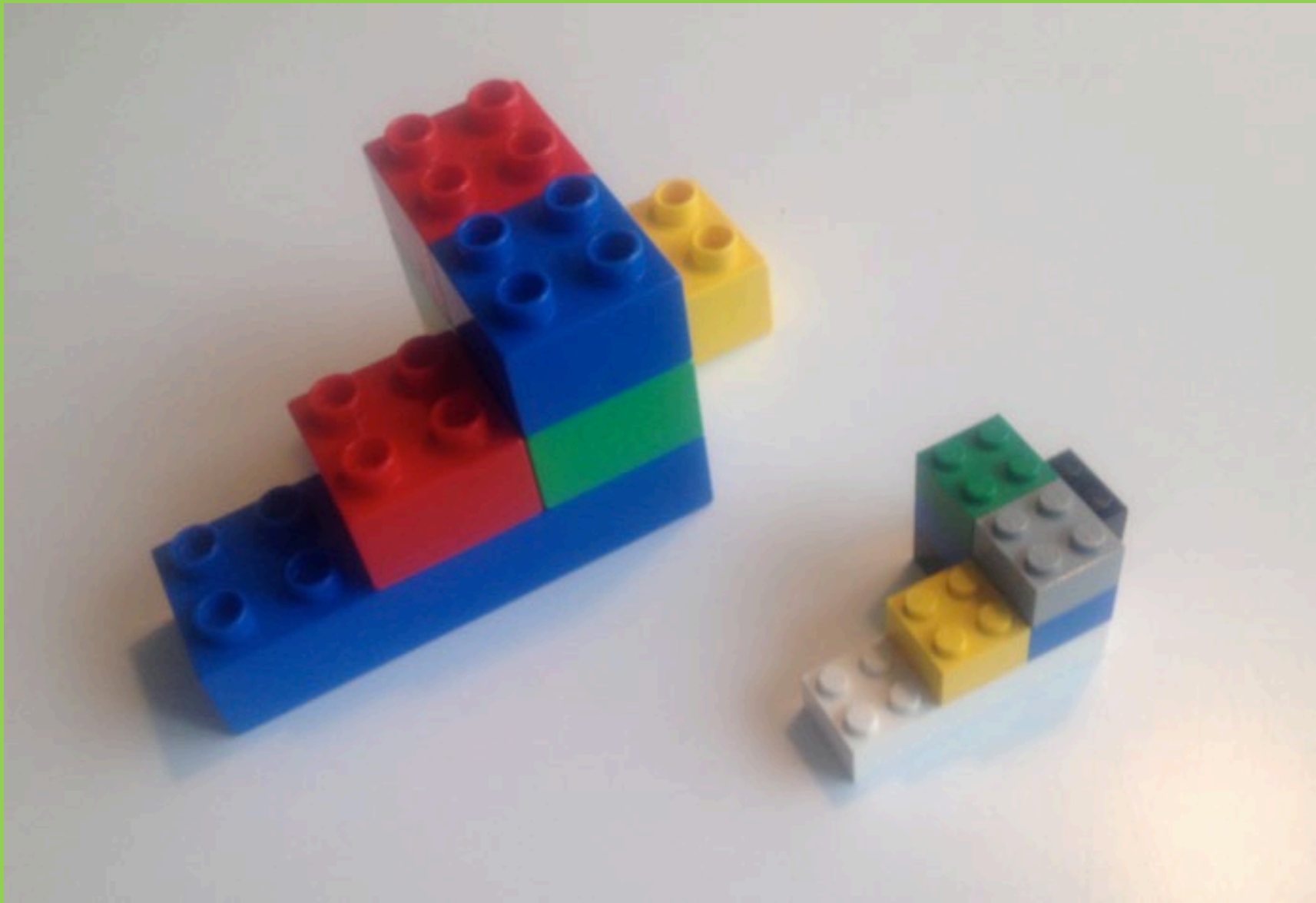
PHOTO





CREATIVITY

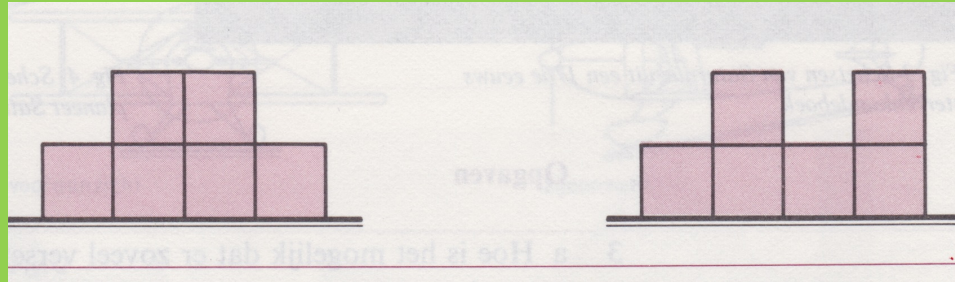




- **LEGO/DUPLO**
- Lots of opportunities
- Similar
- Differences
- Scale
- Linear
- Quadratic
- Cubic
- Top-, Side-, Frontview

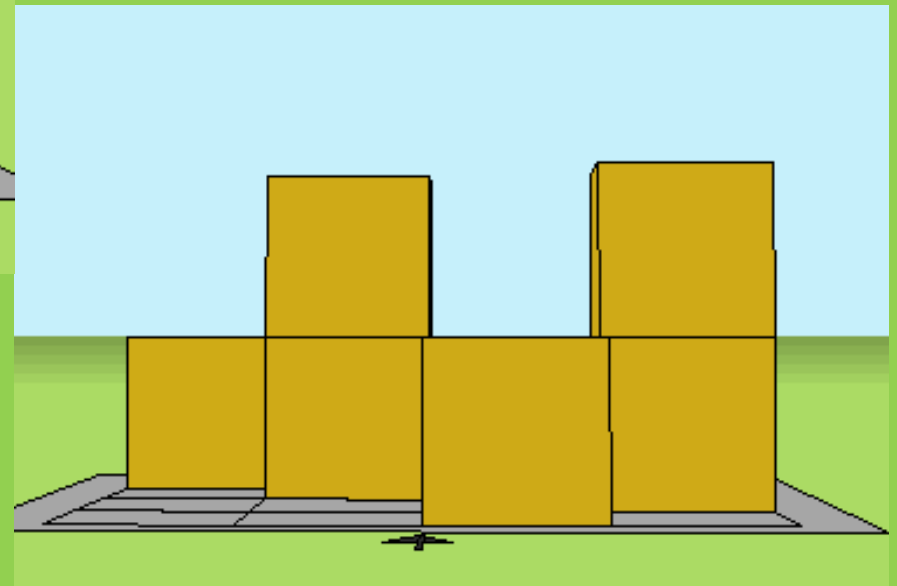
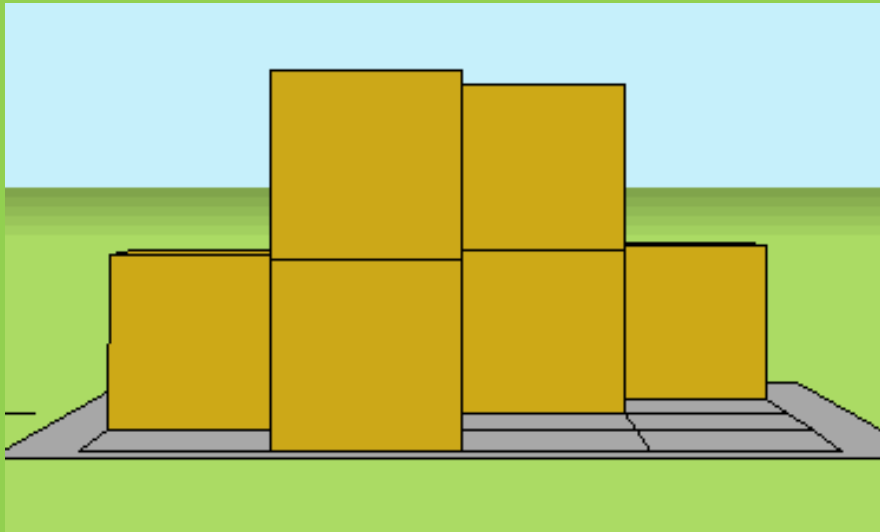
FRONT-AND SIDEVIEW

HOW MANY BLOCKS DO YOU NEED?



Hoeveel blokjes heb je nodig?

FRONT & SIDE VIEW



GEOMETRY

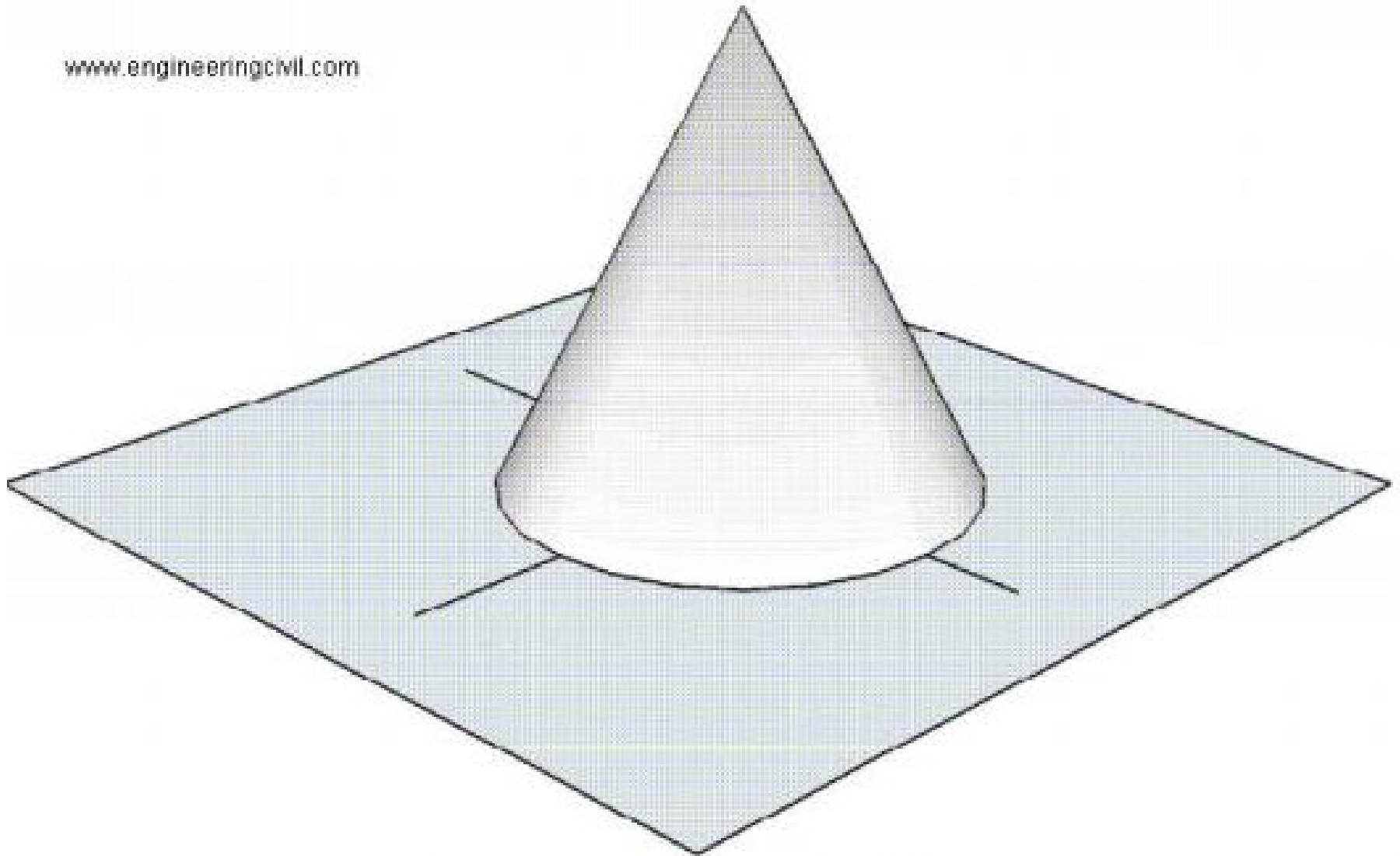
- Geometry is grasping space... that space in which the child lives, breathes and moves.
- The space in which the child must learn to know, explore, conquer, in order to live, breathe, and move better in it

GEOMETRY

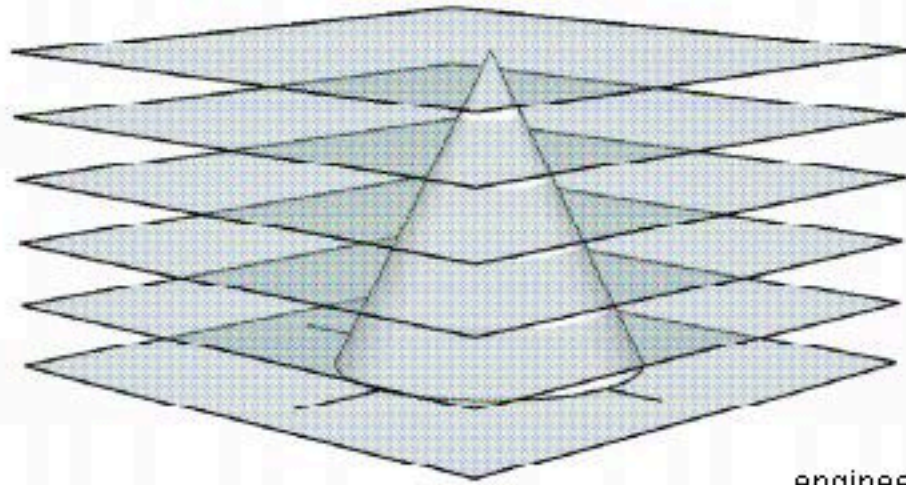
SPACE & SHAPE



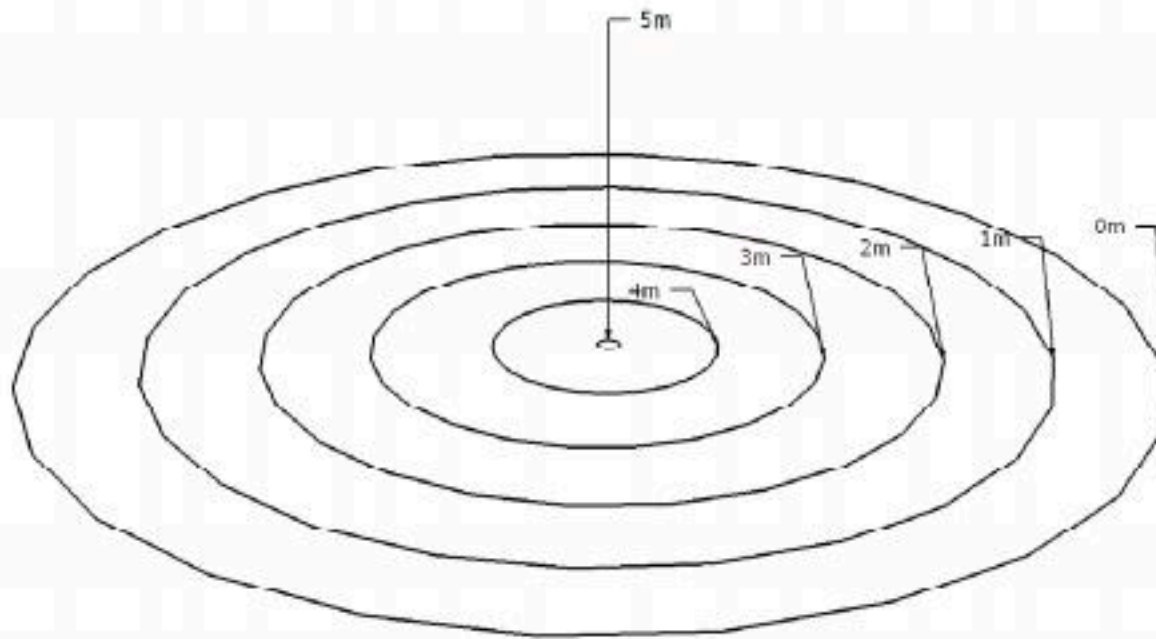




Cone on a 0m level plane



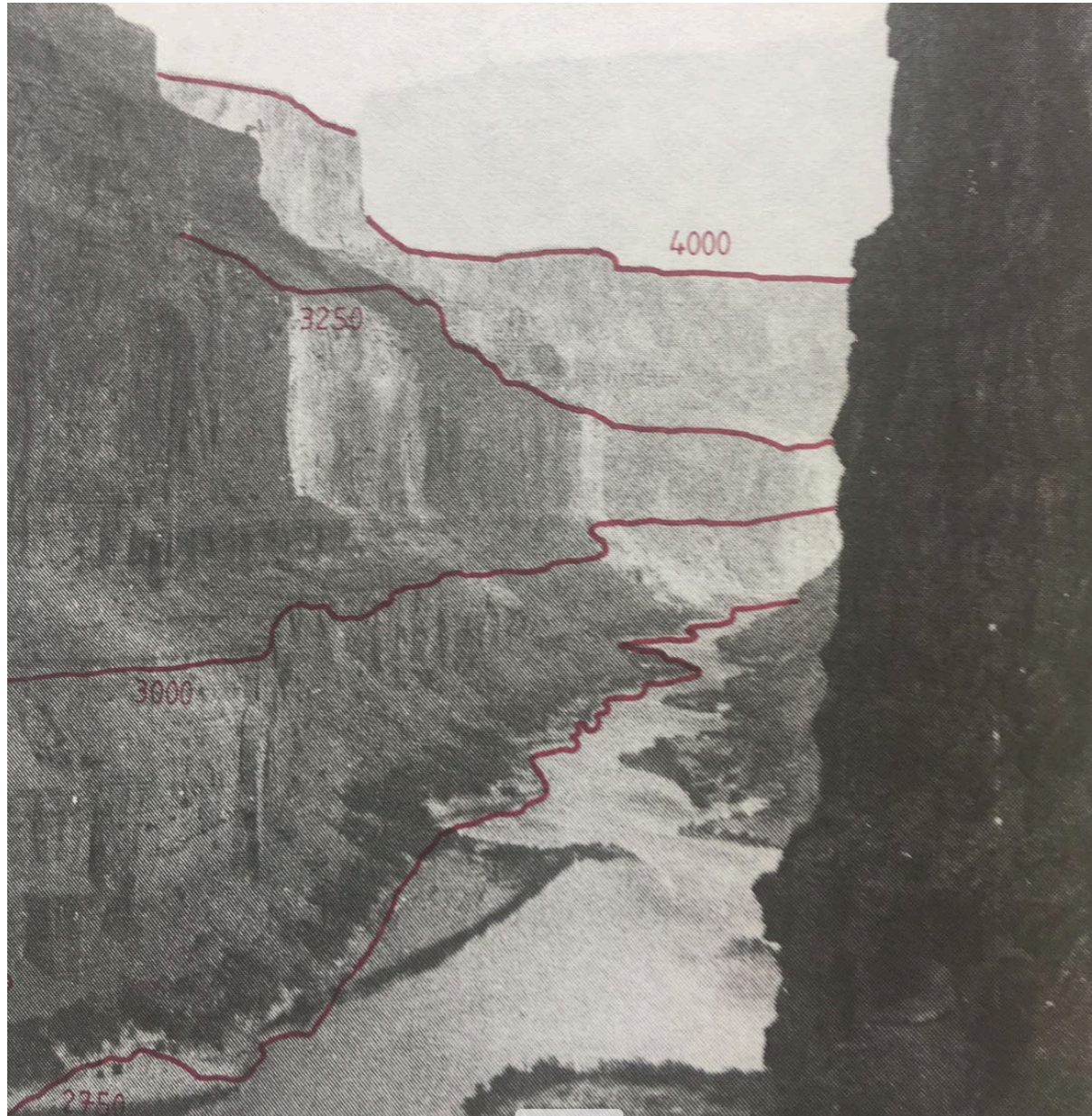
engineeringcivil.com



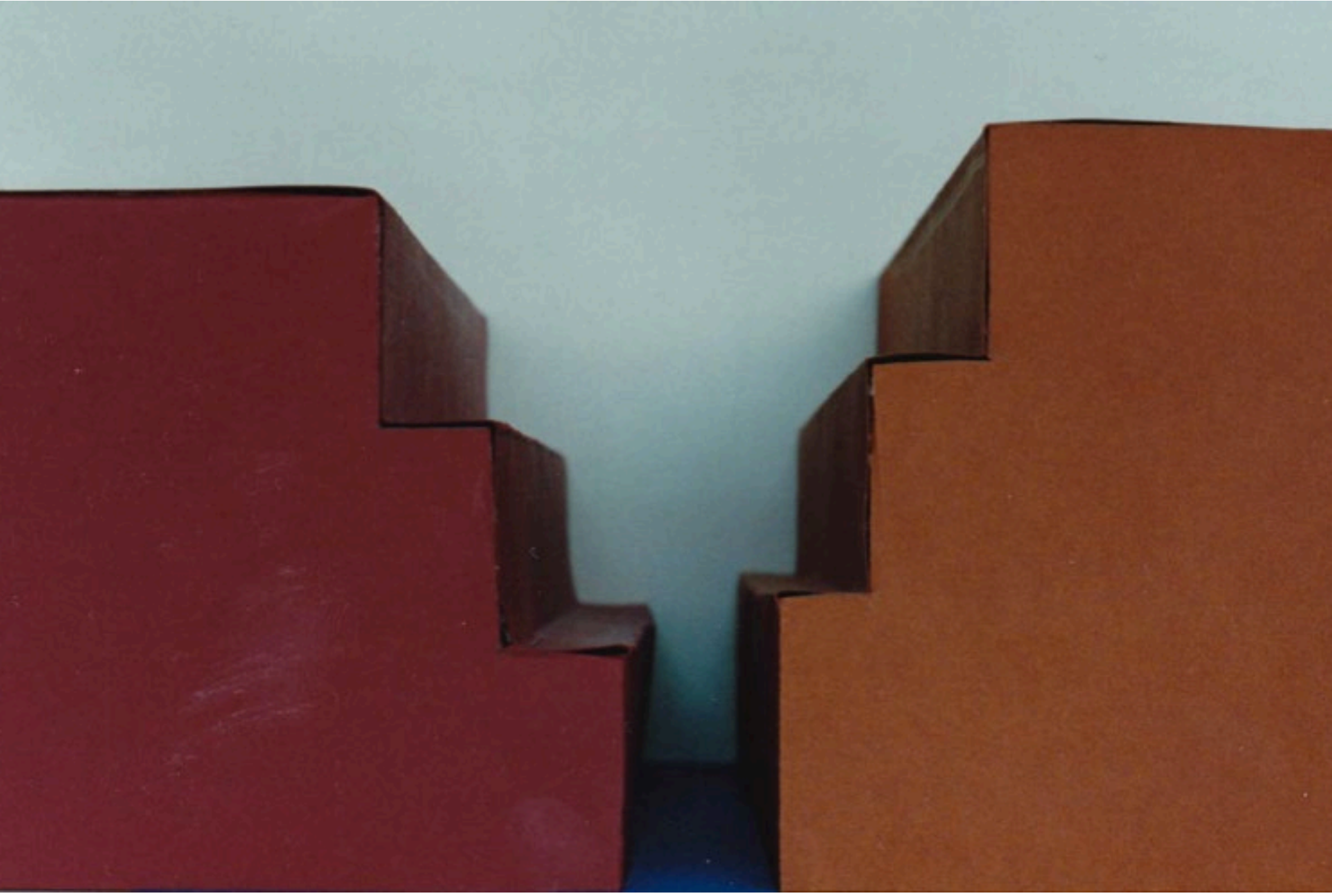
**Cone sliced at 1m intervals (top)
Contour map of the cone drawn to a convenient scale (bottom)**



GRAND CANYON GEOMETRY

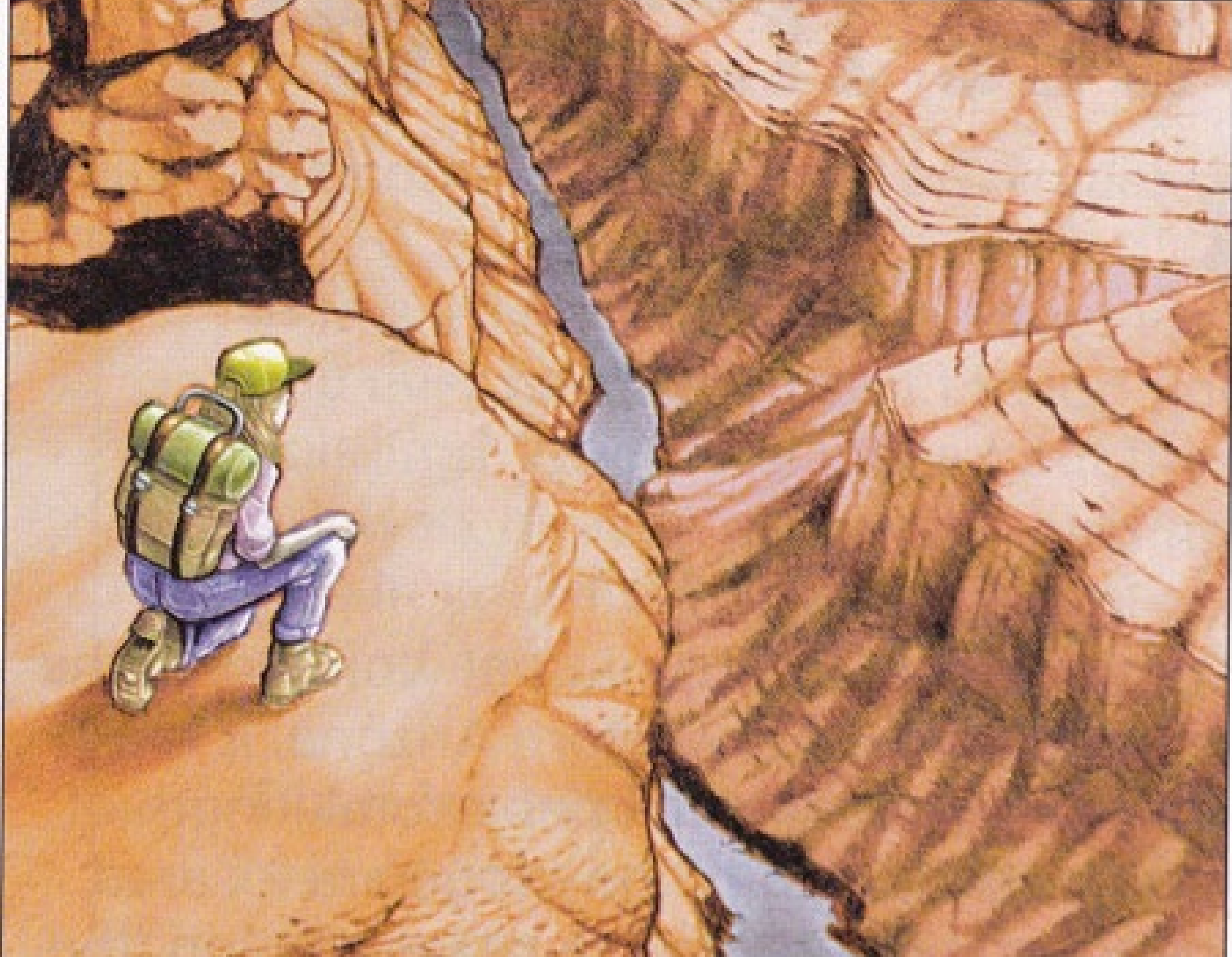


First Model: Concrete



(Re)invention Vision Line (Line of Sight)

- 2^e Model:
- Student Lookout



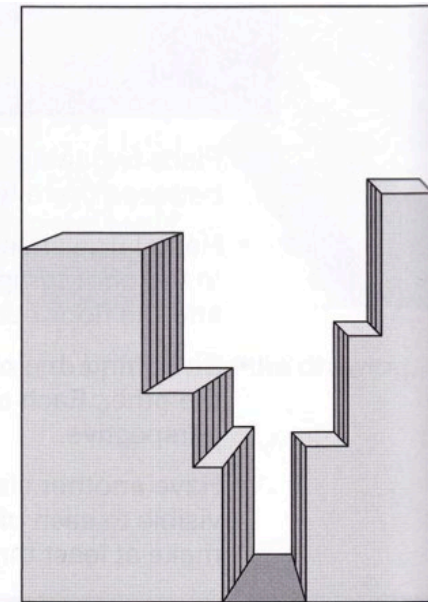
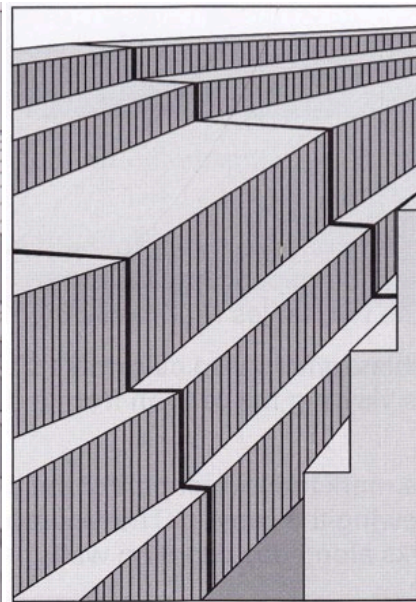
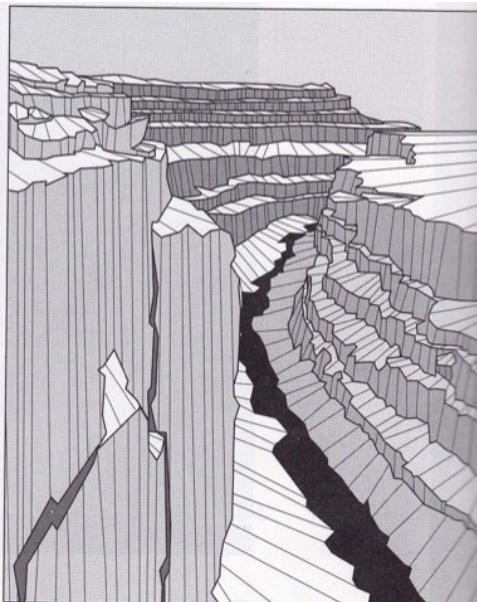
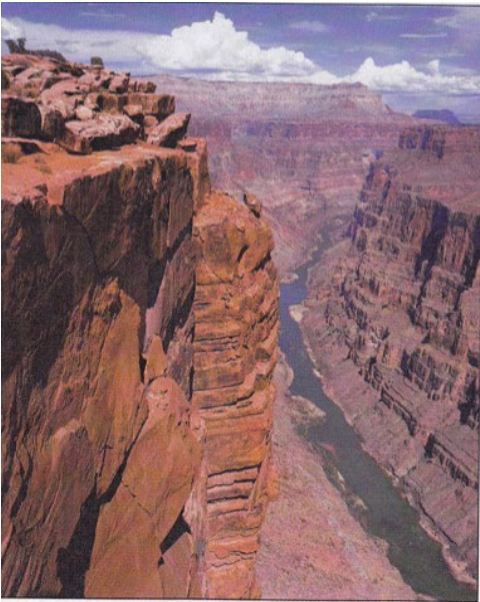
(Re)invention Kijklijn

- 3^e Model:
- Canyon Tables

How deep can you see?



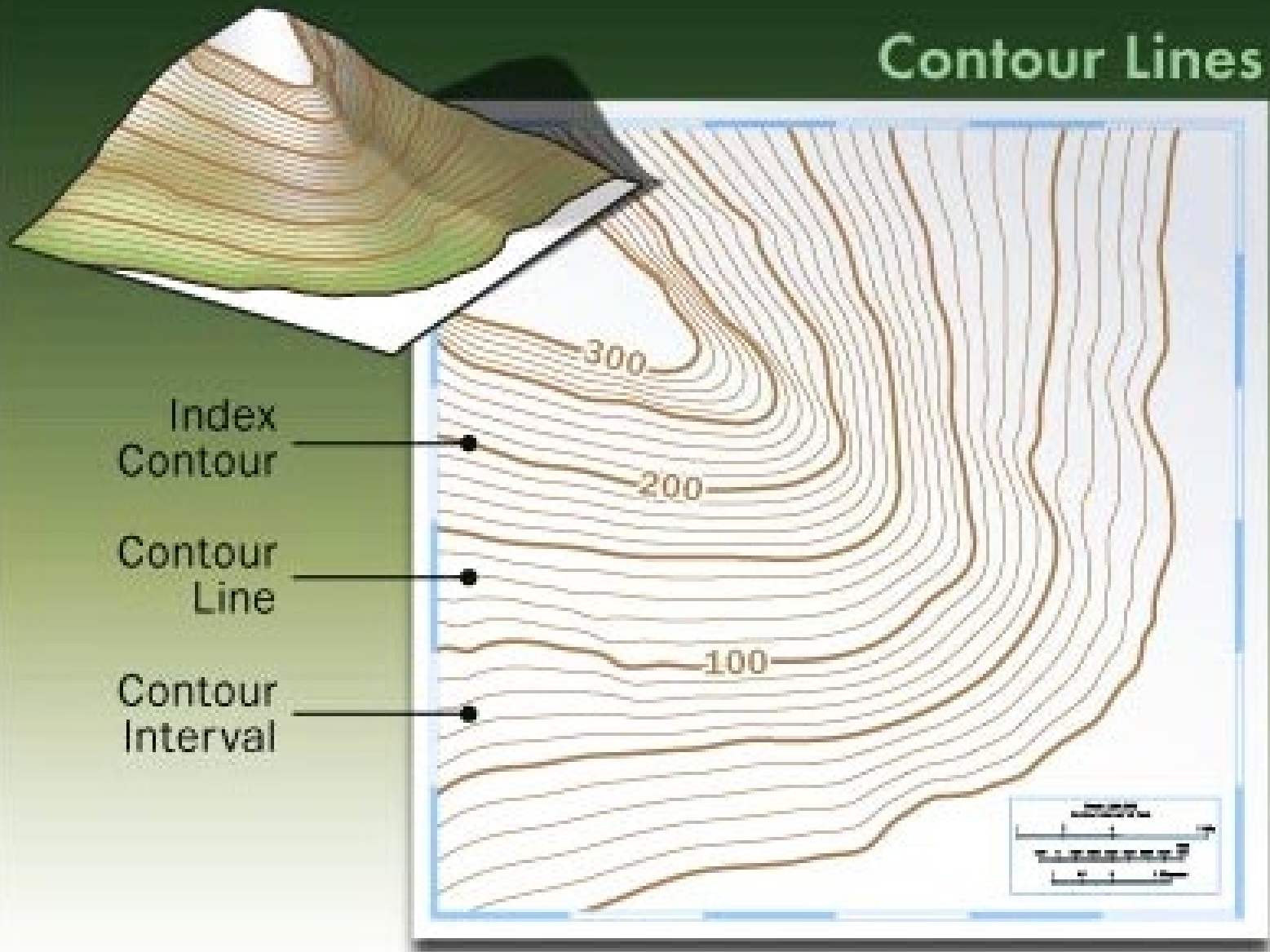
GRAND CANYON

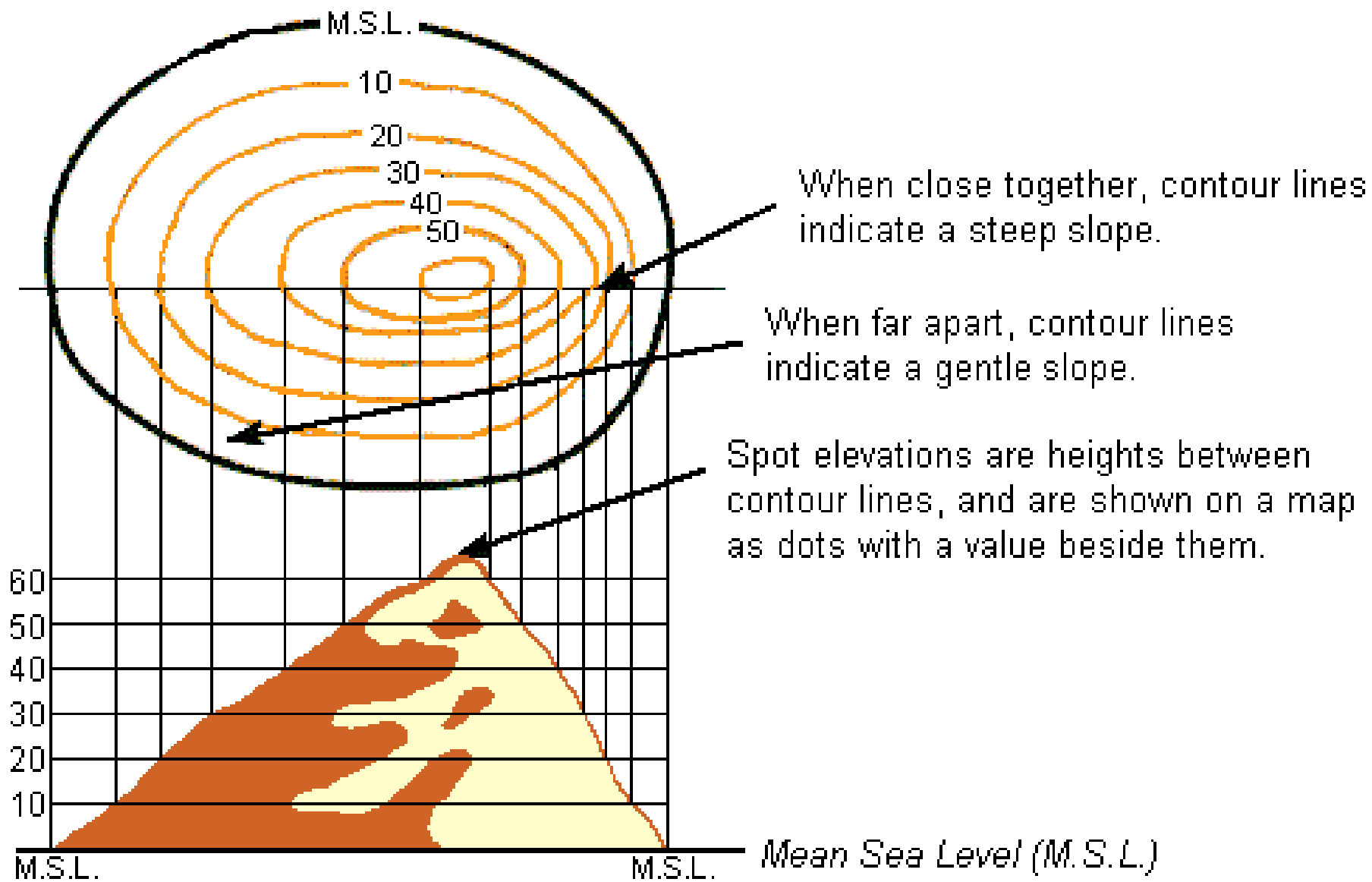


GUNUNG KINABALU



Contour Lines





Op Noord Borneo bevindt zich de 'Berg der doden', ofwel Gunung Kinabalu. Het hoogste punt is 13455 voet en heet Lows Peak, naar de Engelse ontdekkingsreiziger Low. Maar er zijn meer pieken zoals uit het kaartje van fig. 38 blijkt.

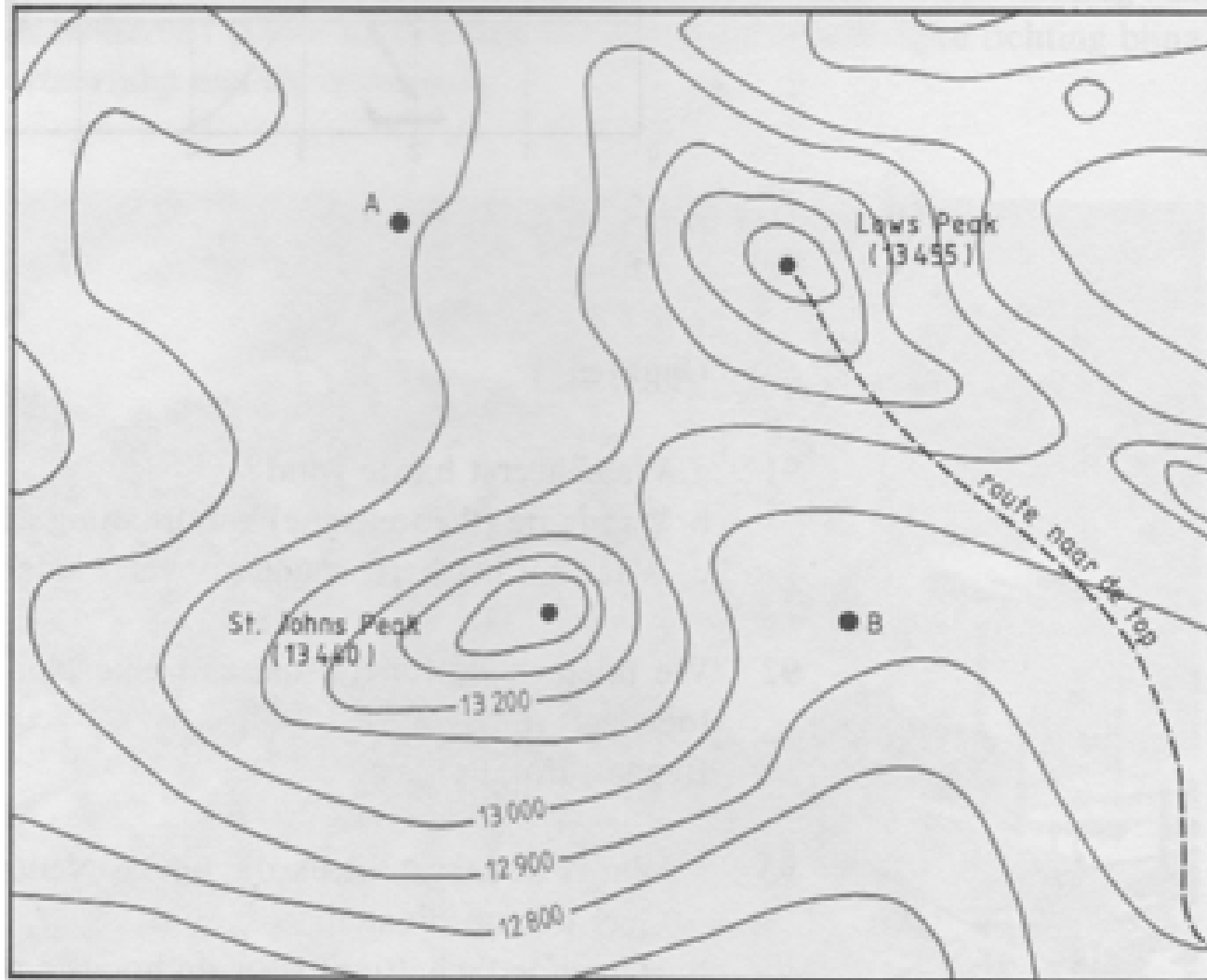
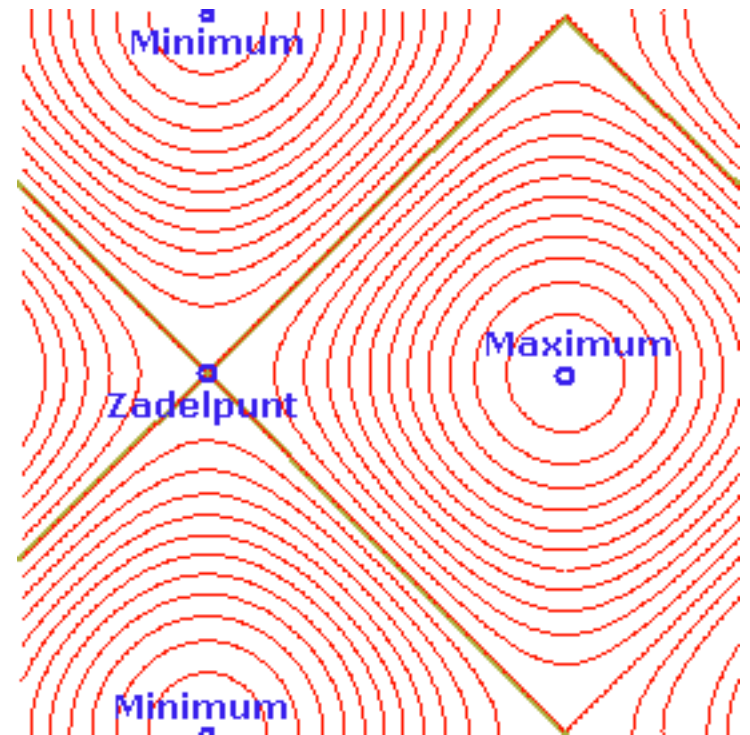
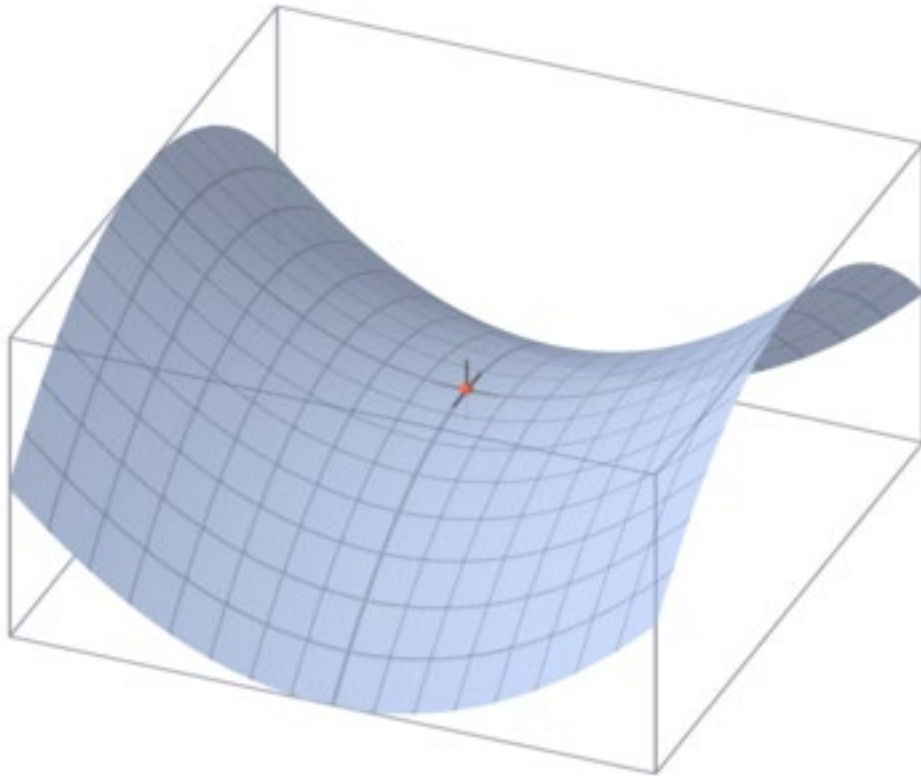


Fig. 38

400 m

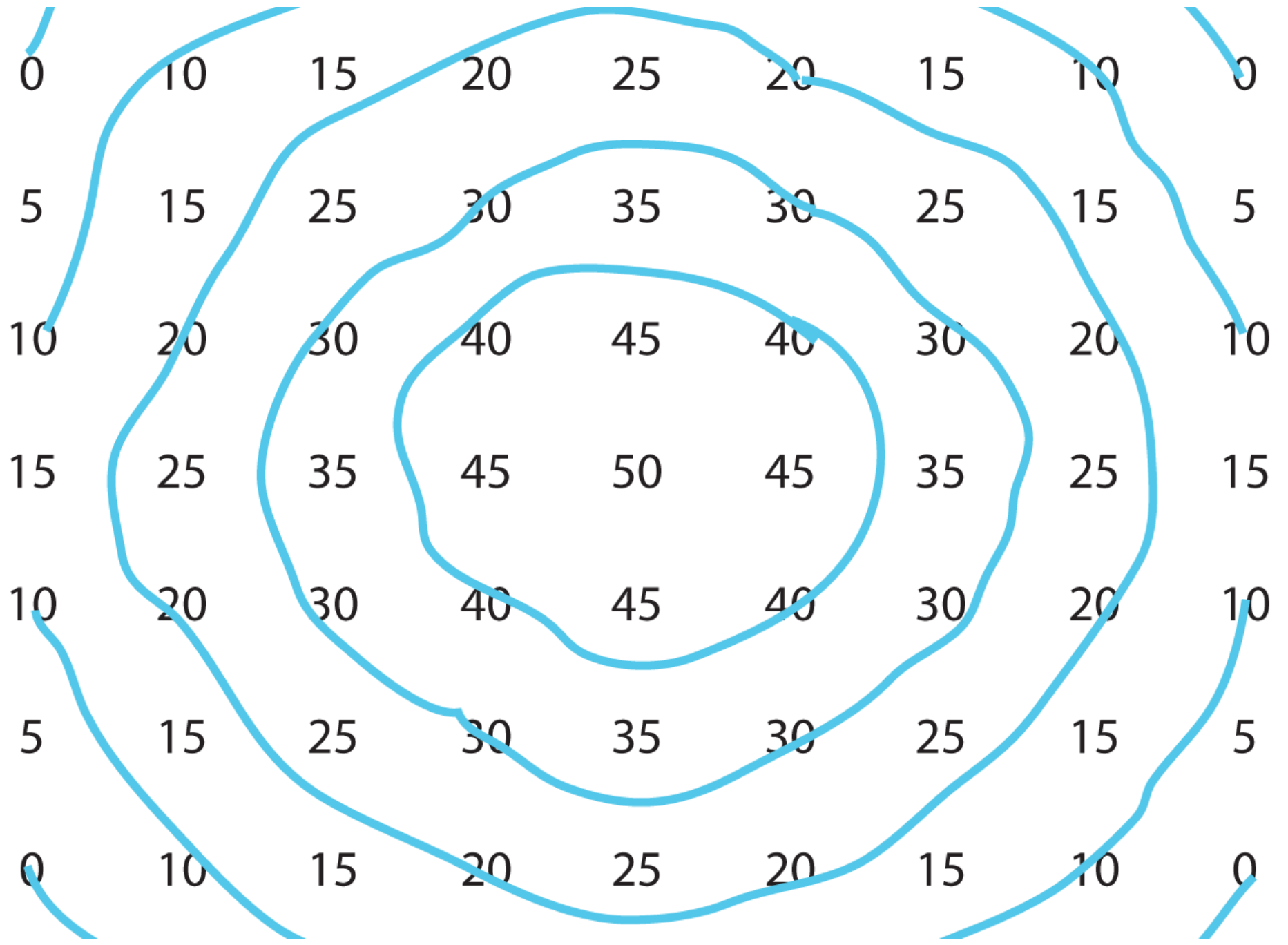
SADDLE POINT

Both maximum and minimum, depending on walking direction



From heights of Points to Contour Lines

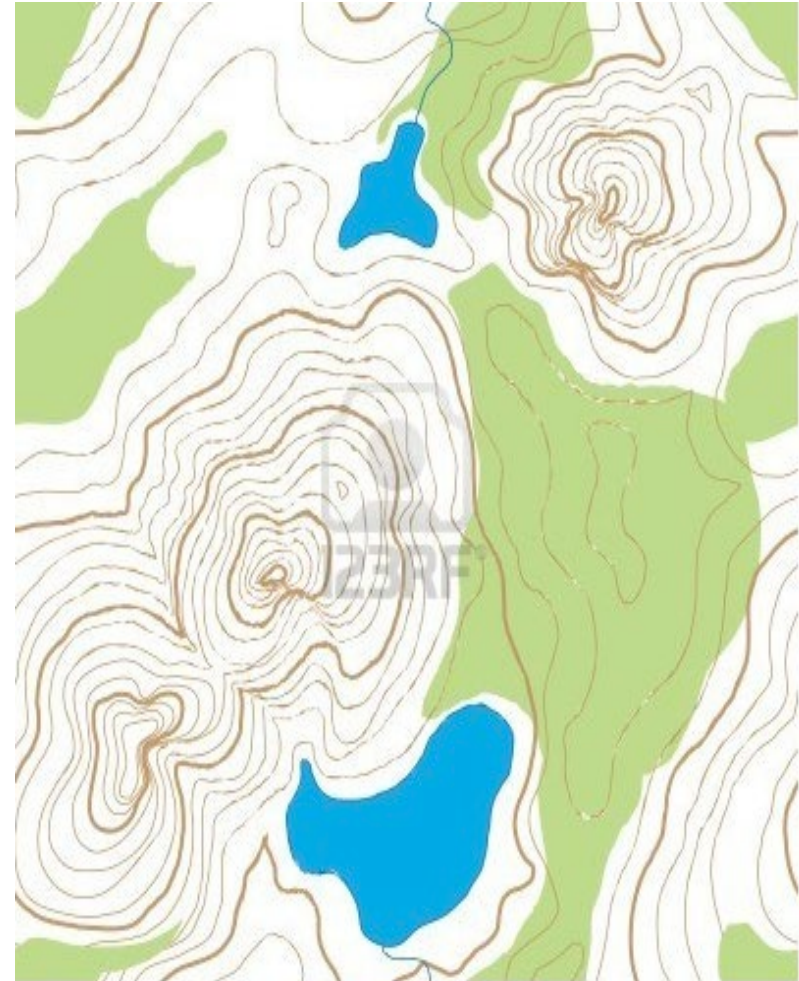
- Connect point with equal heights,
- Make your own contour lines map
- Why can't contour lines intersect?
- Etc.



10	20	30	20	10	0
20	30	40	20	10	0
30	40	45	30	20	10
20	30	40	20	30	10
20	30	20	30	40	30
10	20	30	40	50	40
0	10	30	30	40	30

ANALYZING CHARTS

- Saddle point?
- Where is it STEEP?
- Where are the plains?
- Why do contourlines not go through lakes?



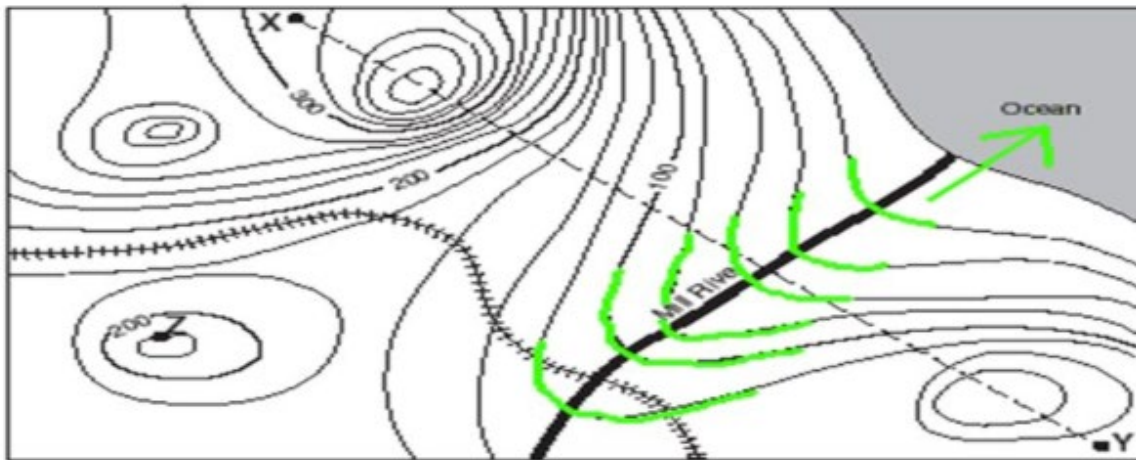
WEATHER MAPS: IT'S ALL THE SAME

From Contour Lines to Isobars: Lines connecting dots with equal air pressure (bars)



The steeper, the faster

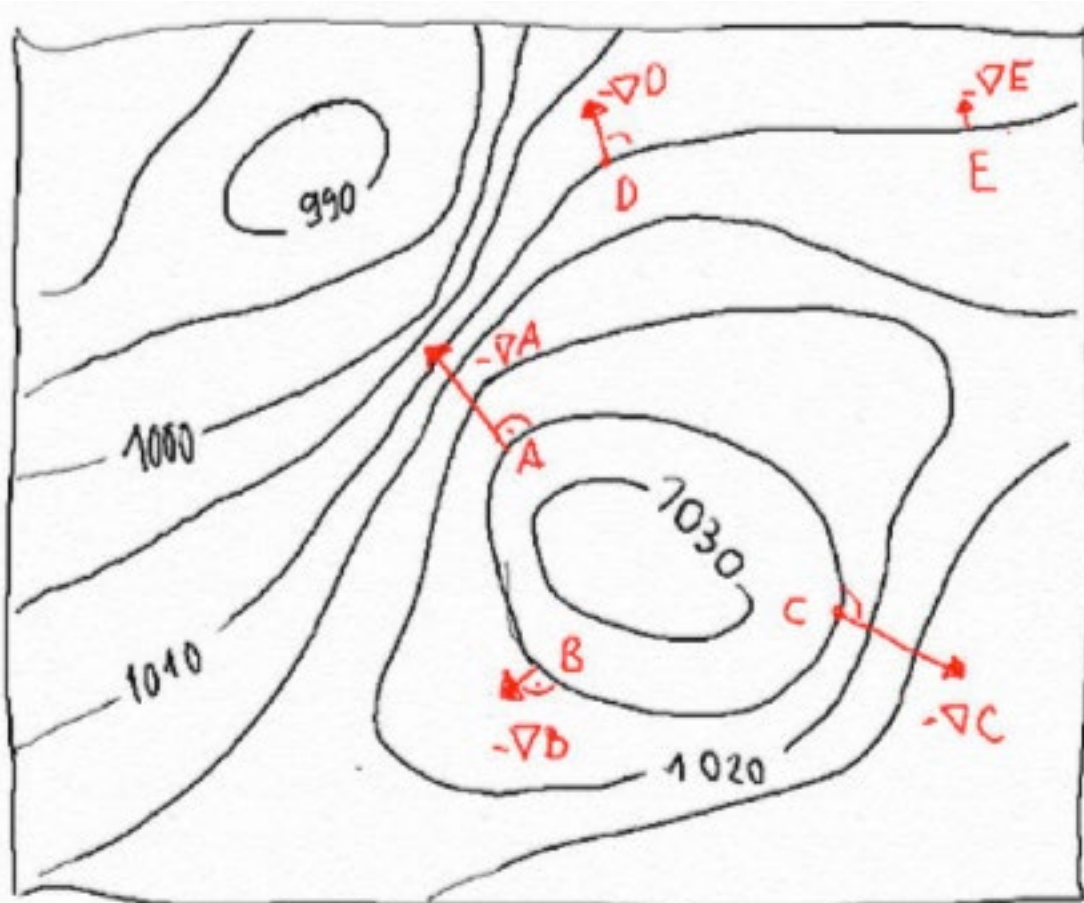
- An object moves perpendicular to contour lines



- In principle this applies to wind as well

Air moves (in Principle -) from H to L (WIND)

On this meteorological map you see the small arrows, indicating the gradient, perpendicular to the contour lines (isobars): the longer the arrow, the steeper the 'hill'



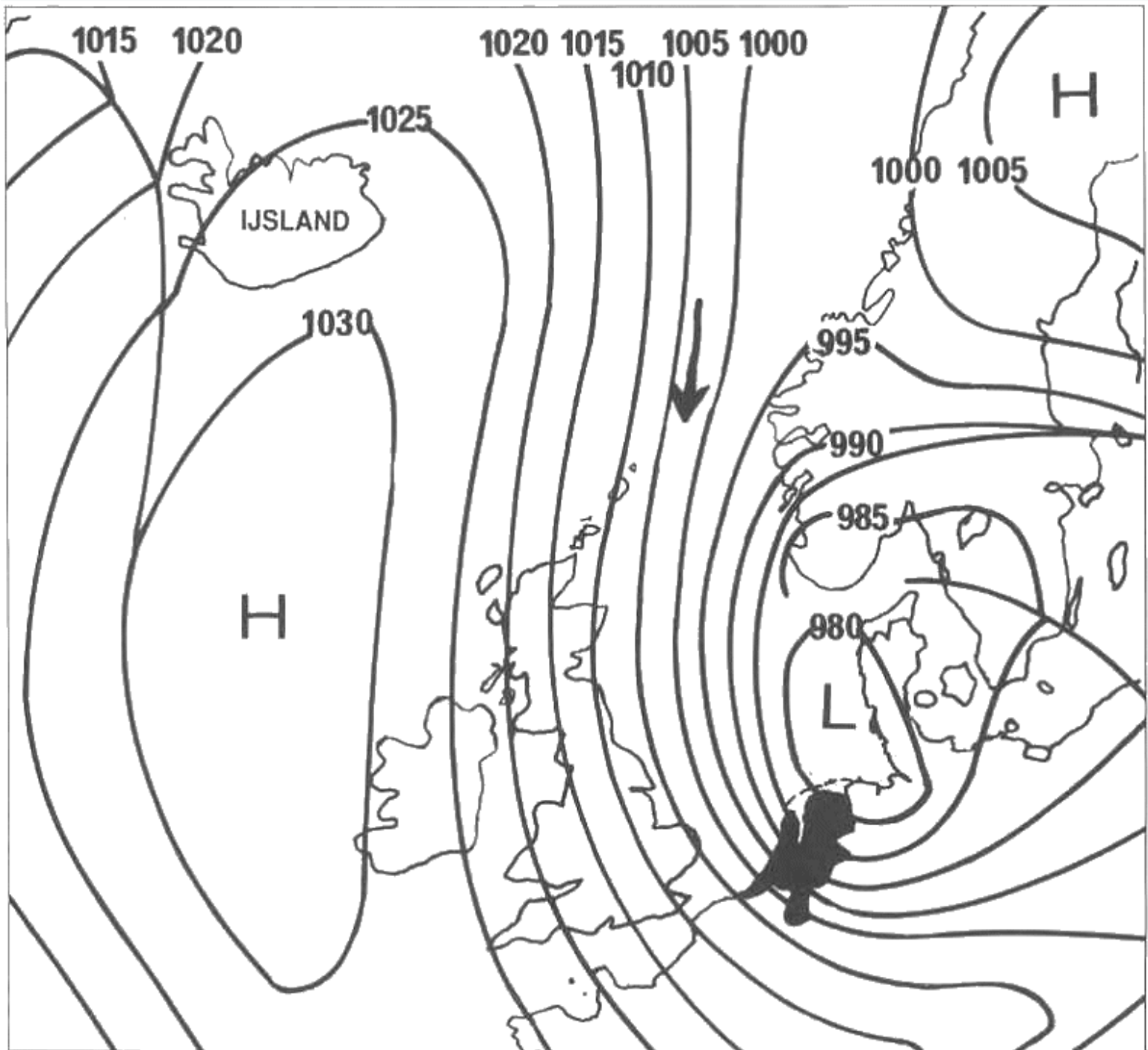
BUT.....

EARTH ROTATION makes life more complex
Storm damage 1 feb. 1953 in Katwijk

And the weather map

Look at wind direction



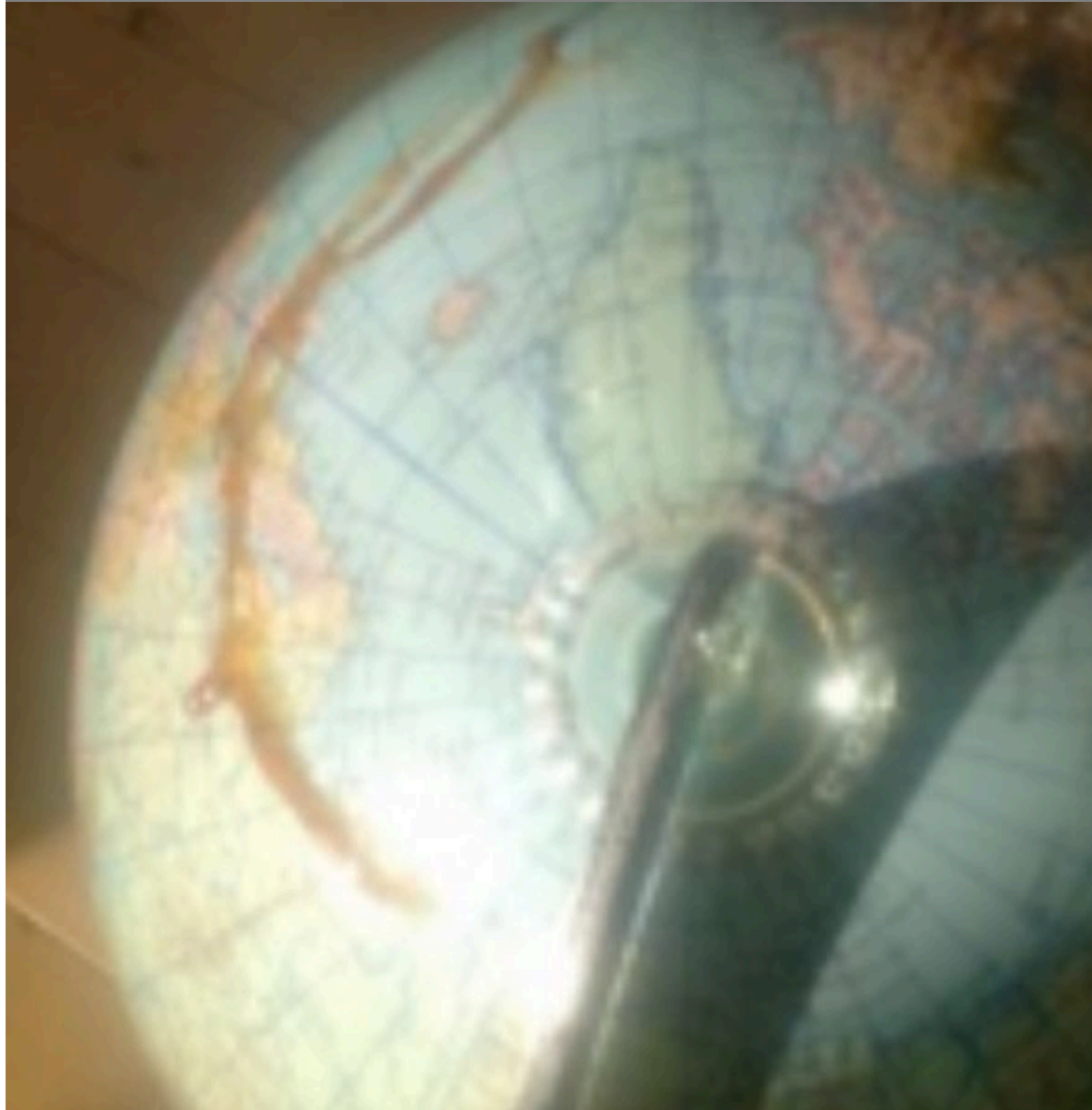


THE EARTH ROTATES

Therefore the wind-direction changes

- Wind almost parallel to iso bars
- Coriolis force
- Experiential proof:

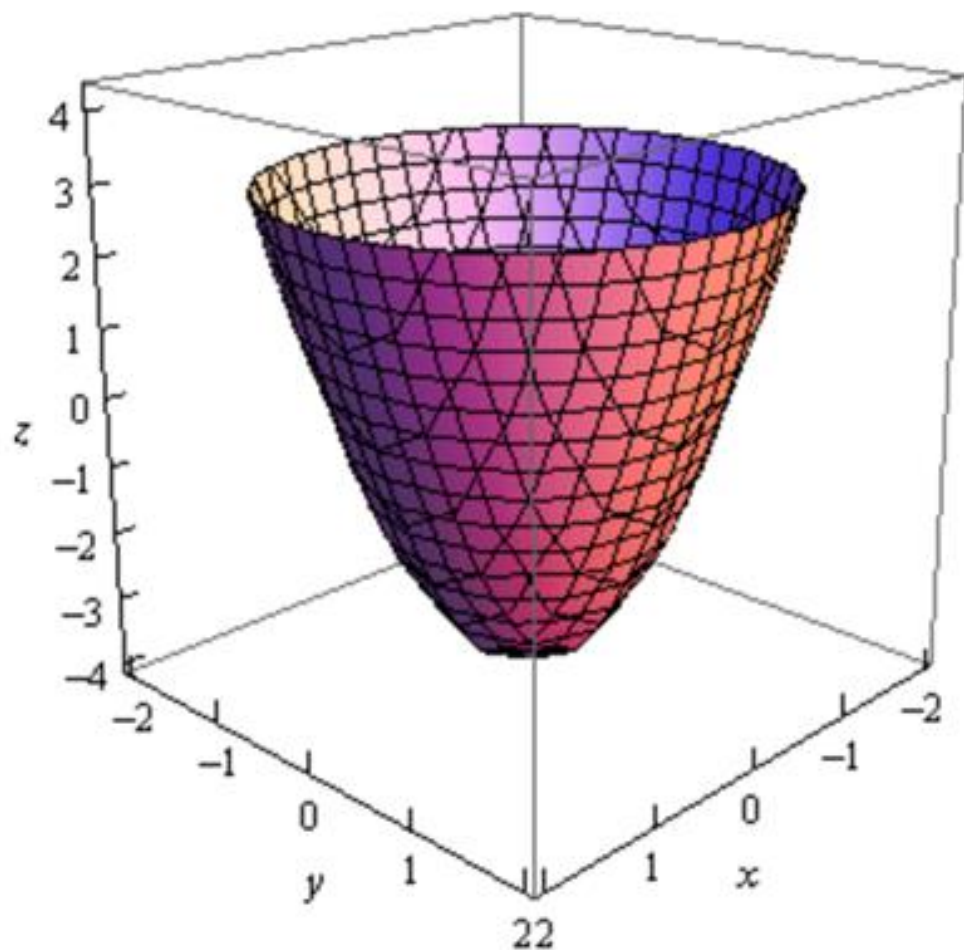
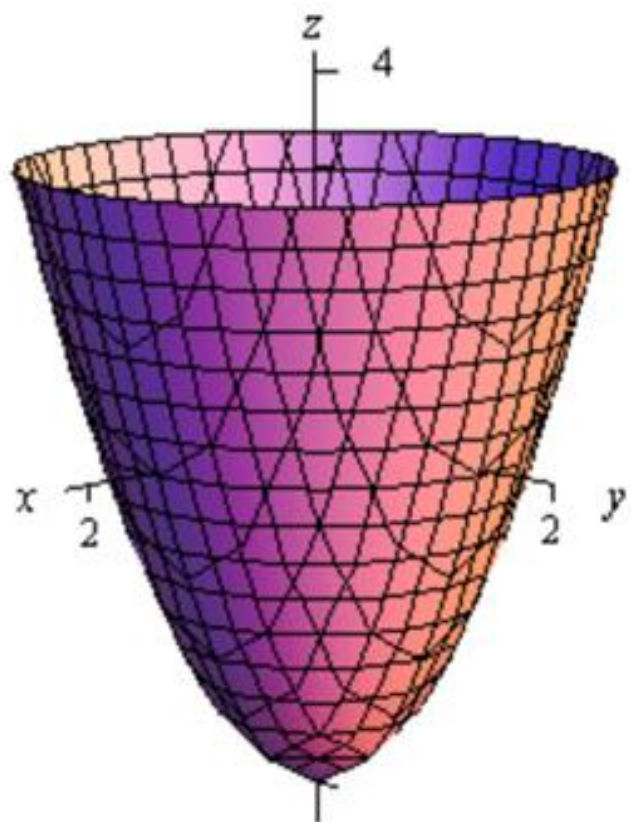
CORIOLIS



Graphs of Functions of 2 Variables

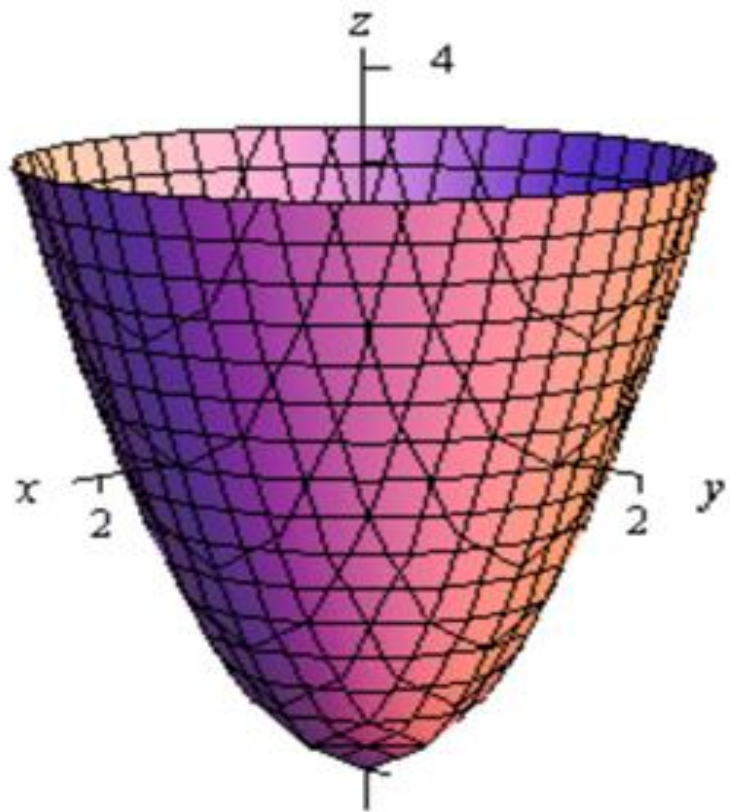
- Can be seen als contour lines, and higher level formal mathematics
- FROM SIMPLE TOY
- TO HIGHER LEVEL MATH

the graph of $z = 2x^2 + 2y^2 - 4$.





the graph of $z = 2x^2 + 2y^2 - 4$.



“NEW”:

CURIOSITY BASED LEARNING

Characteristic

- 1. Real world problem solving (incl: playing)
- 2. Challenging
- 3. Exploration
- 4. Thinking & Doing
- 5. Reflection

REFLECTION: INQUIRY & CURIOSITY BASED PROBLEM SOLVING

Air Squirt -> **Ratio & Proportion**->**Measurement**

T-shirt problem-> **Algebra**

Car size Photo-> **Visual Geometry: Vision lines, perspective, area/volume rule (Lego/Duplo)**

Contour Lines->Slopes->Gradients-> **Functions of 2 Variables**

All using concrete **materials**

CURIOSITY/INQUIRY-PROBLEM SOLVING-INTERACTION

TEACHER INTERACTIONS **minimal**: with **max** effect

REFLECTION

- INQUIRY BASED (better name?):
- **CURIOSITY BASED Learning & Problem Solving**
- Starting with Challenging Problem with concrete materials
- Developing Learning Trajectories, Exploration
- Similar to Realistic Mathematics Education
- **It works..... if you believe in it.**
- More and better problems, including use of IT
- **For ALL students**