Mathematical concepts and procedures in a frame of Multimedia

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Vygotsky underlined the role of language when studying conceptual development. It is through our language that we learn, remember and are able to communicate mathematical concepts and procedures. A concept definition holds both the individuals interpretations and understanding of a concept. The concept definition comprises all the processes, properties, and mental images that the individual links to the concept. The individual's intuitive ideas regarding the concept are included in the structure (Tall & Vinner, 1981).

We surely live in a digital time. Technology is all around us and is strongly affecting the way we receive information and the way we learn. All though this submission is about teaching mathematics, it also indicates what probably will be part of many different subjects in the close future.

The short video clips and files for GeoGebra that is included in this paper are derived from the theories about learning mathematics that mostly Tall (2004) has researched and described. Tall claims that conceptual development may be seen as a change of an individual's concept image. Tall (2004) suggest the model of three mathematical worlds in order to describe the cognitive development that correlates with mathematical understanding.

My video clips, right now translated into several languages and used in Sweden, Finland, Kenya, Tanzania, and other countries, can be exampled by the following short (1 minute 6 seconds) video clip: https://youtu.be/JQuNSt2wK70

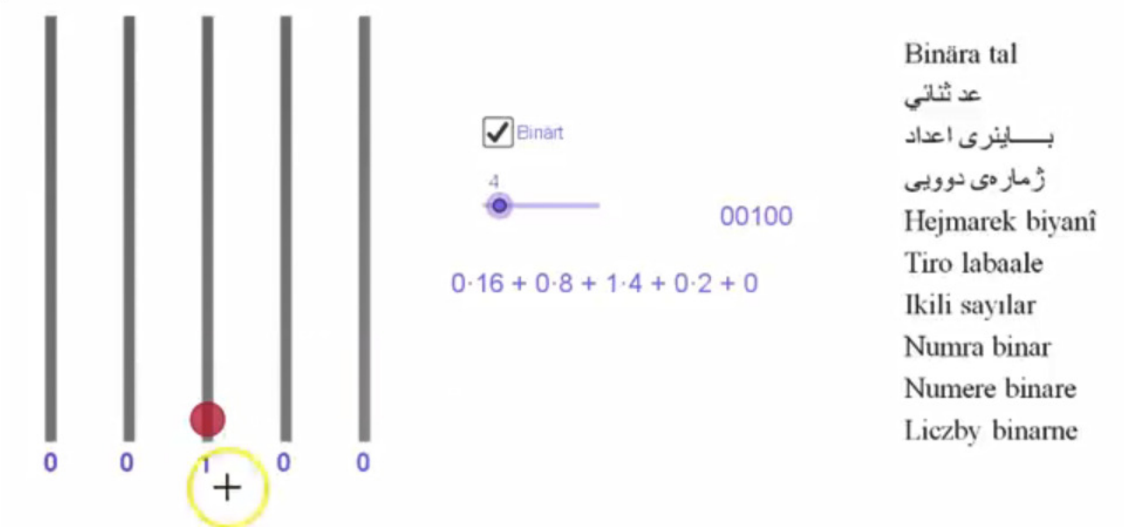

Figure 1: Binary numbers and their relation to decimal numbers.

In figure 1, you see that the concept definition is presented in different languages to the right in the movie. The concept image is intertwined with the procedure shown to the left in the video clip. The video is not intended to exclude any teacher; it is produced to be used by a teacher in some country. It has no sound. It shows how binary and decimal numbers works.

Each video clip address is attached with a short instruction for teachers about how to use the video. In short the instructions address the use of technology such as computer, iPad or smart phones, the way to present the content of the films, when to stop and ask questions and so forth.

Research questions:

How will teachers use these films?

What will students learn from these films?

Three studies have been concluded so far, all of them in Swedish middle schools. I will first present students and teachers voices regarding how they view the mathematics and the language in the films. After that I will show how a multimodal theory has helped me doing the films in a certain way.

Study 1:

A group of Somalian students in 5th grade were given the opportunity to follow lessons given by a research assistant (RA1) in which the content were given by short video clips. Below are two such examples.

The first example is of the concept of area unit. The video show how one of the five shapes are filled with 9 area units and then the viewer of the film is asked how many area units the other shapes will need to be filled. So the viewer is asked to generalize from the first example.

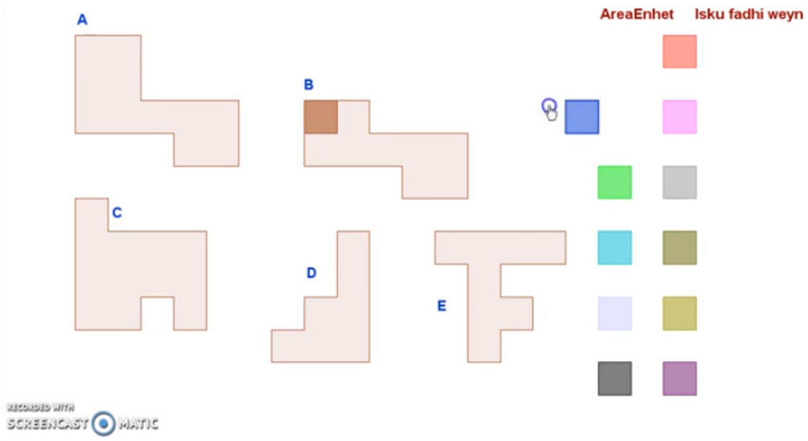

Figure 1: See https://www.youtube.com/watch?v=eu8f3fO9-fQ

The second example show a more complex video where area and perimeter is measured at the same time as the scale is changed.

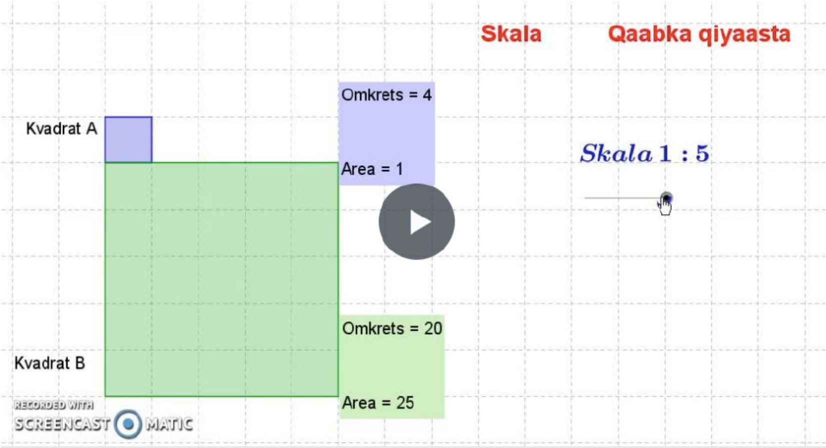

Figure 2: See https://youtu.be/GgnPI_Wvkko

The results from the series of video supported lessons were analyzed with Tall's & Vinner's (1981) concept definitions and concept images and Tall's (2004): Three mathematical worlds. Tall (2004) suggests a possible categorisation of cognitive growth into three worlds of mathematics or three distinct but interacting developments in human thinking.

Three worlds of mathematics are founded on the assumption that the learning of mathematical concepts is individual and develop at different stages, through perception, through symbols or through axioms.

The first world is the conceptual – embodied world, the world we meet through perception, the visual and spatial mathematical world. Most of us have a concept image of what a circle is. A circle is round, it may be large or small and it may be red or blue. We have not learned this through educational efforts; instead we have learned this through the physical world and through observations. The first mathematical world consists of objects we have discovered and observed in the real world, knowledge we have gained through our senses. It also contains mental conceptions of non-existing objects such as point with no size and lines with no thickness.

The second world is the proceptual – symbolic world. In this world we find symbols and actions that we have to perform when we, for example, are dealing with manipulations in algebra. Central in this world is the concept of procept which consist of the first part of process and the end of the word concept. Gray & Tall (1994) introduced the concept procept to describe a central part of the learning of mathematical concepts. Gray et al. (1994) underlined that it is important to learn how to apprehend mathematical symbols both as concepts and as parts of a process at the same time.

An elementary procept is the amalgam of three components: a process which produces a mathematical object, and a symbol which is used to represent either process or object. (Gray et al., 1994,. p. 12)

According to Gray et al., 1994, $2 \cdot 3$ may be perceived as a process (multiplication) or as a concept (product). Regarding learning in the symbolic world of mathematics he/she may use and reflect over the mathematical symbolic language and its function, meaning and application.

The third mathematical world is the formal axiomatic world. Is this world are axioms, theorems and proofs in focus. Based on given assumptions regarding the proportion and relation between mathematical objects are axiom based structures built and used as foundations for mathematical theorems.

Mathematical thinking is thereby based on perception developing subtly in sophistication through the mental world of conceptual embodiment. The development takes account of the individual's previous experience which may operate successfully in one context yet remain supportive or become problematic in another, giving rise to emotional reactions to mathematics, leading to a spectrum of success and failure over the longer term (Tall, 2004).

Results study 1

The results showed that the Somalian students gained mathematical understanding as well as mathematical procedural competence. They were given a pre-test and a post-test and performed better after they had seen the video clips. Also the negative attitudes towards mathematics were replaced with a more positive attitude. That was probably linked to the fact that they saw mathematics described in Somalian words.

> *The student reacts very positive when I tell her that her mother tongue should be incorporated in the teaching. The student asks me if other students from Somalia could be allowed to participate in the lesson and she is so positive that some of the concept definitions are in Somalian. Although the student understands most of the concepts described in Swedish, she tells me that it is so good to be able to check her understanding by the Somalian word ”sometimes I am a little bit unsure and then I can double check the Somalian word and then I know if I am right or wrong”.*

In this dialogue, the teacher is communicating with one of the Somalian students after the students had seen the first video.

Study 2

Seven middle school teachers were asked questions about teaching material and in the interviews they were also asked about some of the video films in a study done with another research assistant (RA2). The research assistant showed four video clips to the teachers and interviewed the teachers about the value of them. Below is one of the video clips that the teachers reviewed and wanted to use.

The Clock is programmed to show both analogue and digital representations of time. Time is also given in Swedish in short sentences as in “A quarter past eleven” since that is, according to teachers, what students find difficult to interpret.

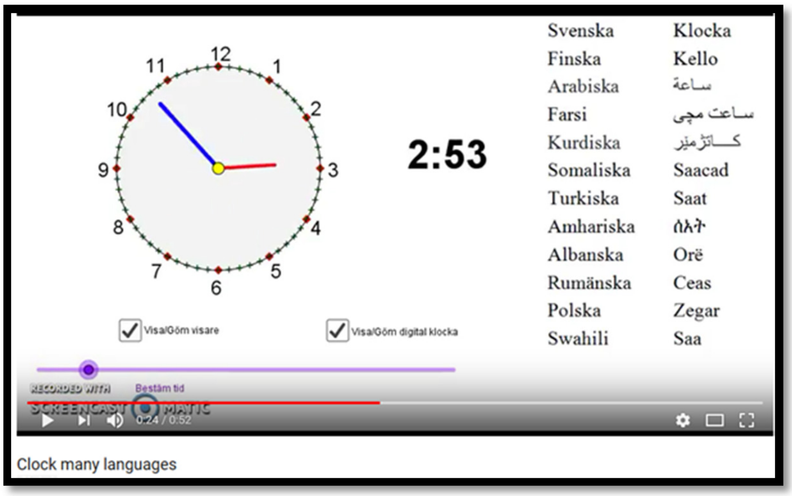

Figure 3: See https://youtu.be/jaLl5XX4A34

Teacher reactions: *That is extremely visual. I would very much be given allowance to use them. I have just been talking about how we measure and communicate time … the video was extremely clear and instructive … I think that the lesson could be more fun when it is multimodality... I also think that it could be so helpful to have videos like this so the students can watch them again and again in their own pace.*

The video clip's quality were analysed with help of the *Cognitive Theory of Multimedia Learning* from Mayer (2014) when the programming took place.

CTML is sprung from a certainty that people learn more deeply from words and pictures than from words alone. This assertion – which can be called the *multimedia principle* – underlies much of the interest in multimedia learning Humans learn more through text and images together than from just text.

Multimedia means that computer information can be represented through audio, video, and animation in addition to traditional media (i.e., text, graphics drawings, and images). Mayer claims that if humans should be able to learn from Multimedia, then it is important that the media is created in the way that fits human's way of thinking and processing information. Then we are learning.

Mayer (2014) claims that the cognitive theory of multimedia learning is based on three basic assumptions about how the human mind works – namely, that the human mind is a dual-channel, limited-capacity, active processing system. The dual-channel assumption is that humans possess separate information processing channels for visually/spatially represented material and auditory/verbally represented material. The relevance of the dual-channel assumption to the cognitive theory of multimedia learning lies in the proposal that the human information processing system contains an auditory/verbal channel and a visual/pictorial channel. When information is presented to the eyes (such as illustrations, animations, video, or on-screen text), humans begin by processing that information in the visual channel; when information is presented to the ears (such as narration or nonverbal sounds), humans begin by processing that information in the auditory channel.

On-screen text may initially be processed in the visual channel because it is presented to the eyes, but an experienced reader may be able to mentally convert images into sounds, which are processed through the auditory channel. Similarly, an illustration of an object or event such as a cloud rising above the freezing level may initially be processed in the visual channel, but the learner may also be able to mentally construct the corresponding verbal description in the auditory channel.

The second assumption is that humans are limited in the amount of information that can be processed in each channel at one time. When an illustration or animation is presented, the learner is able to hold only a few images in the visual channel of working memory at any one time, reflecting portions of the presented material rather than an exact copy of the presented material. This is called the *limited-capacity assumption*.

The third assumption is that humans actively engage in cognitive processing in order to construct a coherent mental representation of their experiences. These active cognitive processes include paying attention to relevant incoming information, organizing incoming information into a coherent cognitive structure, and integrating incoming information with other knowledge. In short, humans are active processors who seek to make sense of multimedia presentations. This is called *active processing assumption* (Mayer, 2014).

According to Mayer (2014), for meaningful human learning to occur in a multimedia environment, the learner must engage in five cognitive processes:

Process	Description
Selecting words	Learner pays attention to relevant words in a multimedia message to create sounds in working memory
Selecting images	Learner pays attention to relevant pictures in a multimedia message to create images in working memory
Organizing words	Learner builds connections among selected words to create a coherent verbal model in working memory
Organizing images	Learner builds connections among selected images to create a coherent pictorial model in working memory
Integrating	Learner builds connections between verbal and pictorial models and with prior knowledge

The five cognitive processes may be in different orders, and must not be in the order given above. But, in order for learning to take place, the learner must be able to deal with all the five processes (Mayer, 2014). Each of the five processes in multimedia learning is likely to occur many times throughout a multimedia presentation. The processes are applied segment by segment rather than to the message as a whole.

To use dynamic animation in the teaching of mathematics is deeply beneficial for a deeper understanding with the learner. Animation is a form of graphical representation that Mayer (2014) calls *stimulation motion picture.* Animation is a tool for learning according to Mayer & Moreno (2002) which claims that there are seven different principles for how animation should be used inside CTML:

1. *Multimedia Principle* – The learner learn more animation and storytelling than from plain story telling. To combine image and storytelling makes it possible for the learner to find relations between words and images. If just one multimodal way is presented, then the learner must make the other way mentally by himself.

2. *Spatial Contiguity Principle* – Deeper learning when corresponding text and animation are presented near rather than far from each other on the screen Is the text far from the animation, then the learner will waist energy on searching for relations.
3. *Temporal Contiguity Principle* – Deeper learning when corresponding narration and animation are presented simultaneously rather than successively
4. *Coherence Principle* – Deeper learning when extraneous narration, sounds, and video are excluded rather than included.
5. *Modality Principle* – Deeper learning from animation and narration than from animation and on-screen text.
6. *Redundancy Principle* – Deeper learning from animation and narration than from animation, narration, and on-screen text.
7. *Personalization Principle* – Deeper learning when narration or on-screen text is conversational rather than formal (pp. 93-97).

Let us analyse the clip of the clock above according to these seven principles. The dynamic image shows the clock in analogous mode and with a corresponding important word, c*lock*. The word clock is placed near the animation which makes the clip true to principle 2, the *Spatial Contiguity Principle*. The word clock is visualized at the same time as the clock itself, which makes it true to principle 3, the *Temporal Contiguity Principle*. In the video clip, there is only a one animation and one concept presented and the sound is excluded, some the clip is also following principle 4, the *Coherence Principle*. Finally is also the clip true to principle 6, the *Redundancy Principle*, since there are few details on the video.

Conclusion: To score on principles 2, 3, 4, & 6 is quite good.

Study 3

In study 3 there were a mix of students speaking, and reading, Arabic, Dari, Farsi, Kurdish, and Somalian. They watched the films area units, clock, and scale.

Concerning the video clip about area units, the results indicate that the students in this study learned more mathematics by simply watching and discussing. A form of conceptual development could be observed. The student's interest and commitment as regarding mathematics increase when a concept was presented this way. The discussion regarding the concept area unit that occurred in the group was surprisingly intense and long. Students with weak Swedish benefitted the most from this discussion since all students in the group discussed the concept rather freely. The video clip's construction also encouraged the students to compete with each other's when trying to "see" how many area units that were needed to fill out another form.

These students were also given pre-test and post-test and performed better after they had seen the video clips. In the classroom a deep and eager discussion about how to say area in Arabic compared to how to say it in Dari and Farsi also occurred. Also the negative attitudes towards mathematics were replaced with a more positive attitude. That was probably linked to the fact that they saw mathematics described in words related to their mother tongue..

Discussion

We are now living in a true digital time. Today smart phones are spreading at an incredible speed over the world, and today tablets and the Internet are becoming the way many children enter the digital world from their early ages. We today speak of "digital natives" for qualifying our students, who spent so much time playing electronic games, sending SMS and communicating through social networks. The French philosopher Michel Serres calls them "Petits poucets" and "Petites poucettes" due to their capacity to send SMS using their thumbs (Serres 2012). New educational practices have emerged such as the so-called MOOC (Massive Open Online Courses) and they quickly spread out in the university world.

We can no longer deny that we live in a digital era, which affects our personal and social life, our working and learning modes. We can no longer consider that teaching mathematics with technology is just an option. In this changing world, I have found a way to program mathematical situation and I am lucky to know people from many different countries who can help med with translations. There are ways to access my video clips on computers, iPads and on smart phones. At present time this seems to be a fruitful way to go. How can we best use it and what more is needed?

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