

BUILDING THINKING CLASSROOMS



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THE STORY OF JANE

A black and white photograph of a cat chasing a mouse. The cat is in the foreground, looking towards the mouse. The mouse is running away from the cat. The background is a plain, light-colored surface.

If 6 cats can kill 6 rats in 6 minutes, how many cats are required to kill 100 rats in 50 minutes?

- Lewis Carroll

If 6 cats can kill 6 rats in 6 minutes, how many cats are required to kill 100 rats in 50 minutes?

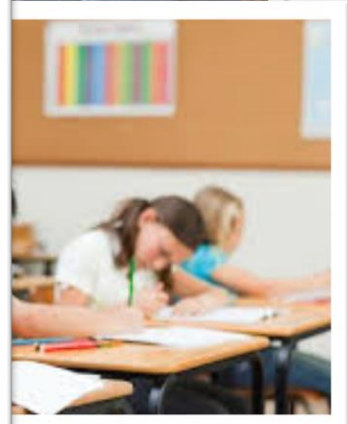
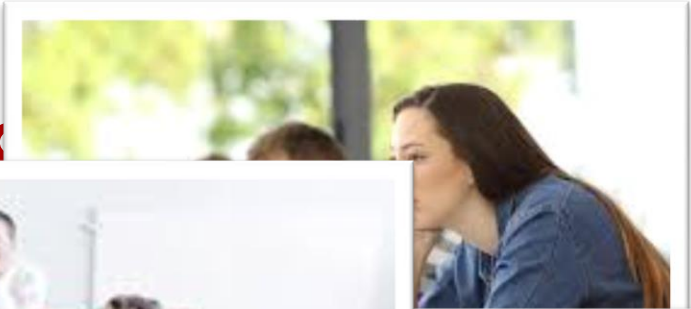
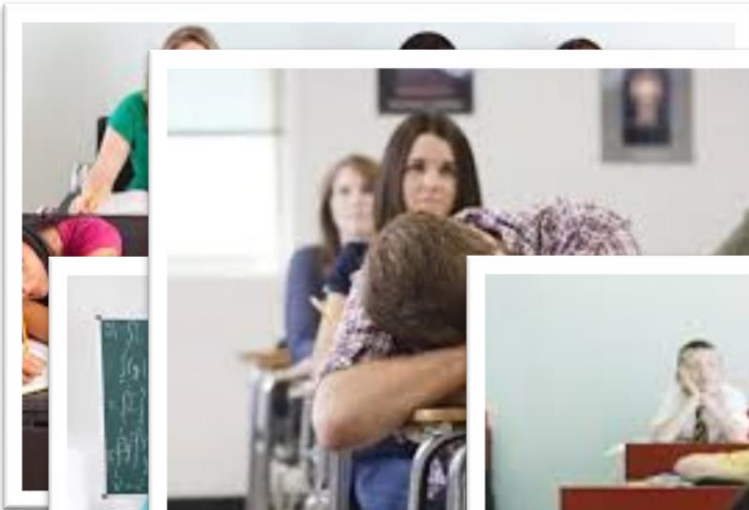
- *Lewis Carroll*

DISASTER!

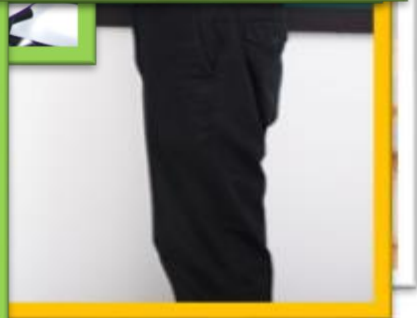
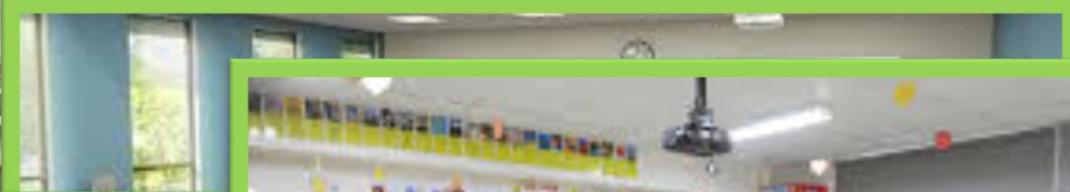
VISITED 40 CLASSROOMS

- good teachers
- snowballing
- elementary + secondary
- English + French
- low socioeconomic + high socioeconomic
- public + private

PATTERNS



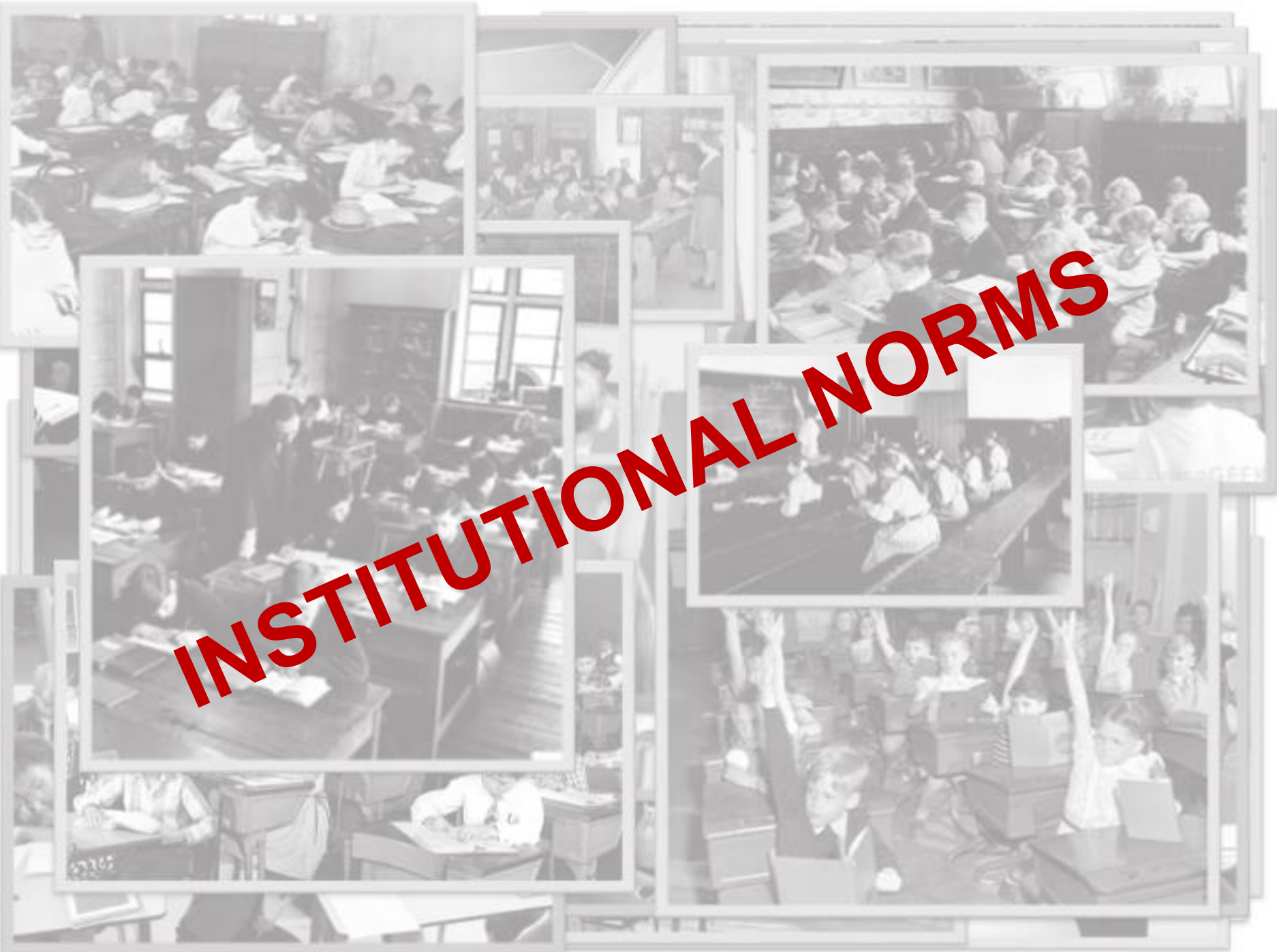








INSTITUTIONAL NORMS



NON-NEGOTIATED NORMS





400+ TEACHERS | 15 YEARS | 2 WEEK CYCLES

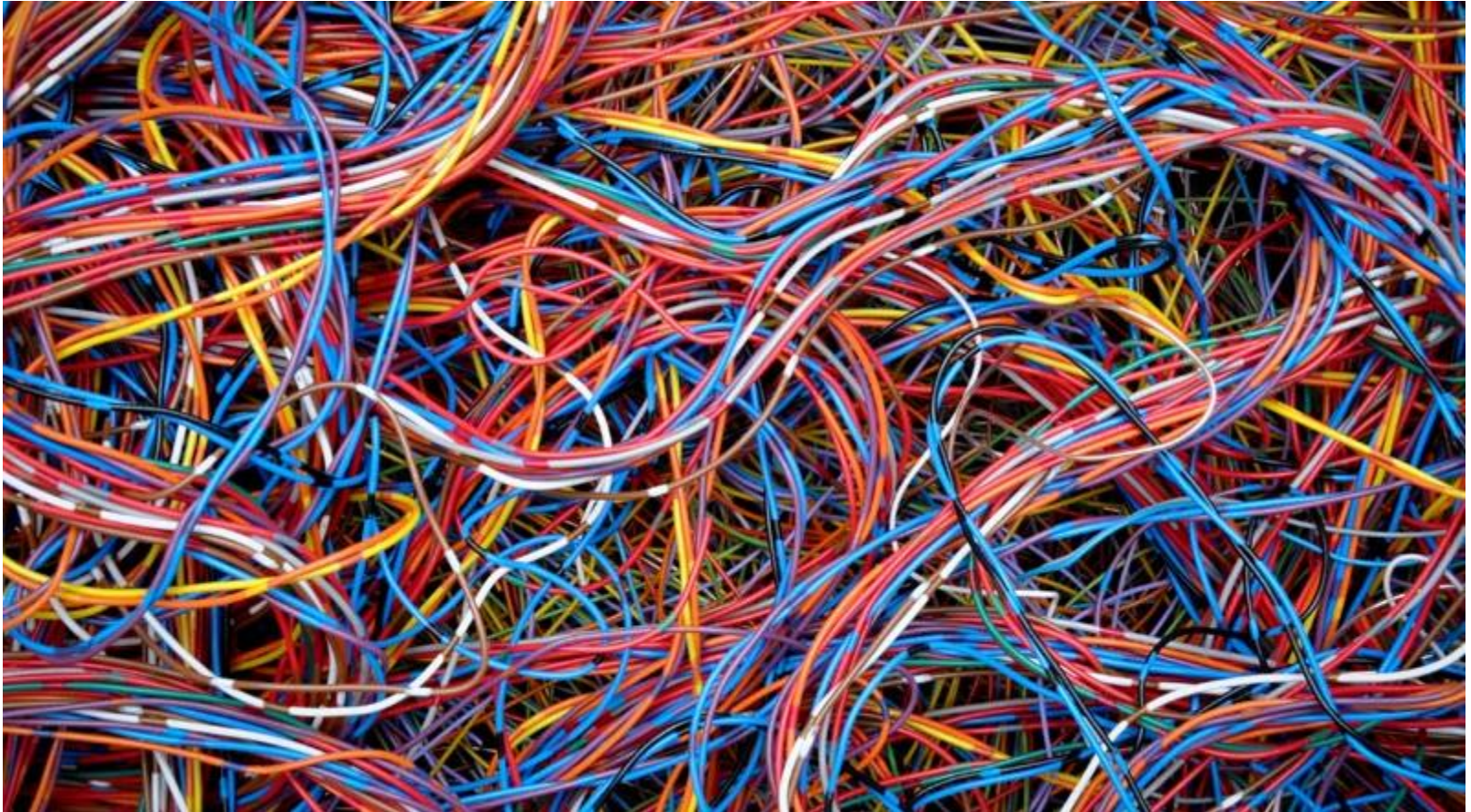


**RENEGOTIATING THE
NON-NEGOTIATED NORMS**

400+ TEACHERS | 15 YEARS | 2 WEEK CYCLES



RAPID PROTOTYPING



CHAOS EVERYWHERE

DIMENSIONS OF PRACTICE

1 tasks

2 when, where, and how we
give the task

3 how we answer questions

4 room organization

5 how groups are formed

6 student work space

7 autonomy

8 how we give notes

9 what homework looks like

10 hints and extensions

11 how we consolidate

12 formative assessment

13 summative assessment

14 reporting out

DIMENSIONS OF PRACTICE***OPTIMAL PRACTICES FOR THINKING***

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DIMENSIONS OF PRACTICE**OPTIMAL PRACTICES FOR THINKING**

1 tasks

thinking tasks

2 when, where, and how we give the task

in the first 4 minutes, students standing in a loose cluster, and verbally

3 how we answer questions

answer only *keep thinking questions*

4 room organization

defront the classroom

5 how groups are formed

frequently form visibly random groups

6 student work space

use vertical non-permanent surfaces

7 autonomy

foster autonomous actions

8 how we give notes

have students do *meaningful notes*

9 what homework looks like

use *check your understanding* questions

10 hints and extensions

manage *flow*

11 how we consolidate

consolidate from the bottom

12 formative assessment

show where they are and where they are going

13 summative assessment

evaluate what you value

14 reporting out

report out based on data (not points)

DIMENSIONS OF PRACTICE**OPTIMAL PRACTICES FOR THINKING**

1	problems	thinking tasks
2	when, where, and how we give the task	in the first 4 minutes, students standing in a loose cluster, and verbally
3	how we answer questions	answer only keep thinking questions
4	room organization	defront the classroom
5	how groups are formed	frequently form visibly random groups
6	student work space	use vertical non-permanent surfaces
7	autonomy	foster autonomous actions
8	how we give notes	have students do <i>meaningful notes</i>
9	what homework looks like	use <i>check your understanding</i> questions
10	hints and extensions	manage <i>flow</i>
11	how we consolidate	consolidate from the bottom
12	formative assessment	show where they are and where they are going
13	summative assessment	evaluate what you value
14	reporting out	report out based on data (not points)

HOW WE GIVE A TASK

textbook

board/screen

handout/worksheet

HOW WE GIVE A TASK

board/screen

handout/worksheet

textbook

HOW WE GIVE A TASK

verbally

board/screen

handout/worksheet

textbook

HOW WE GIVE A TASK

verbally

Tax Man is played like this: Start with a collection of paychecks, from \$1 to \$12. You can choose any paycheck to keep. Once you choose, the tax collector gets all paychecks remaining that are factors of the number you chose. The tax collector must receive payment after every move. If you have no moves that give the tax collector a paycheck, then the game is over, and the tax collector gets all the remaining paychecks. The goal is to beat the tax collector.

Example:

Turn 1: Take \$8. The tax collector gets \$1, \$2 and \$4.

Turn 2: Take \$12. The tax collector gets \$3 and \$6 (the other factors have already been taken).

Turn 3: Take \$10. The tax collector gets \$5.

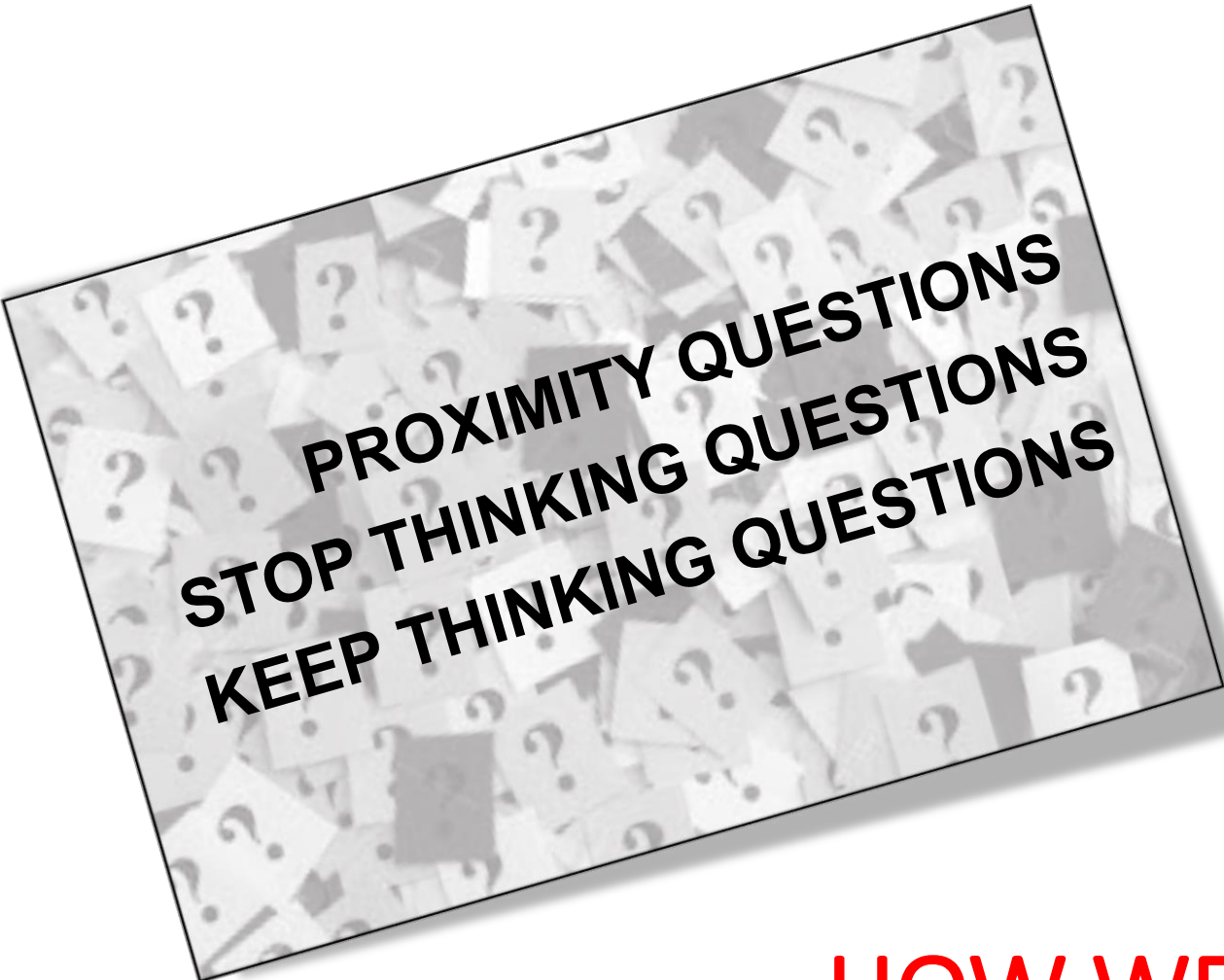
You have no more legal moves, so the game is over, and the tax collector gets \$7, \$9 and \$11, the remaining paychecks.

Your Total Scores:

You: $\$8 + \$12 + \$10 = \30 .




HOW WE ANSWER QUESTIONS



**PROXIMITY QUESTIONS
STOP THINKING QUESTIONS
KEEP THINKING QUESTIONS**

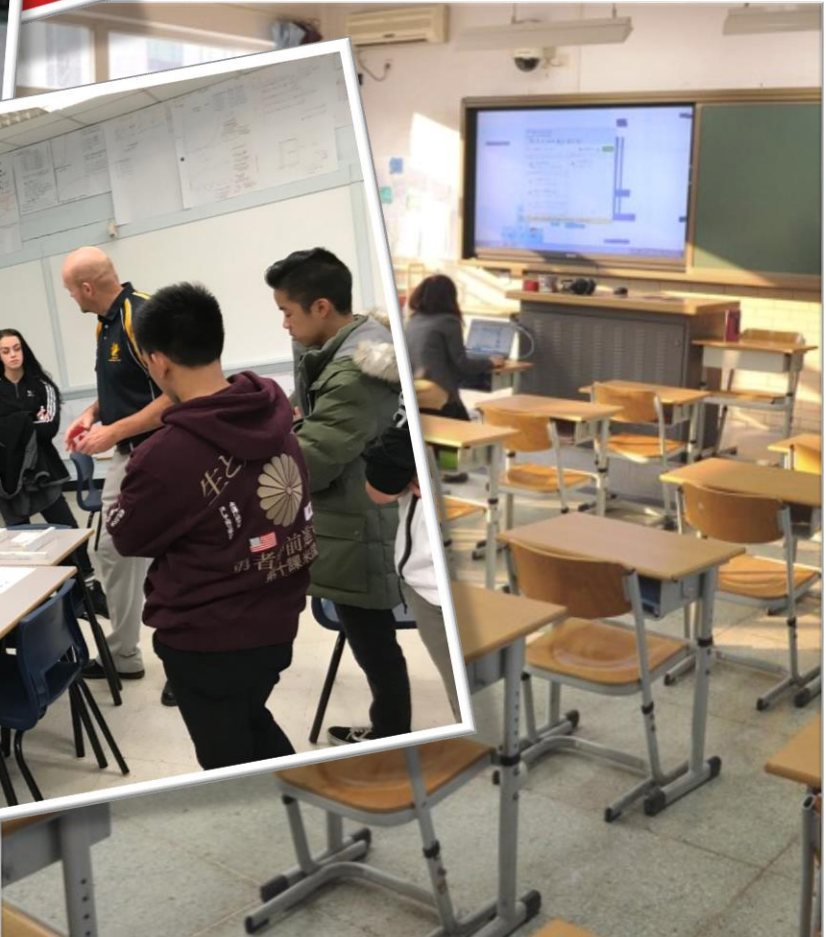
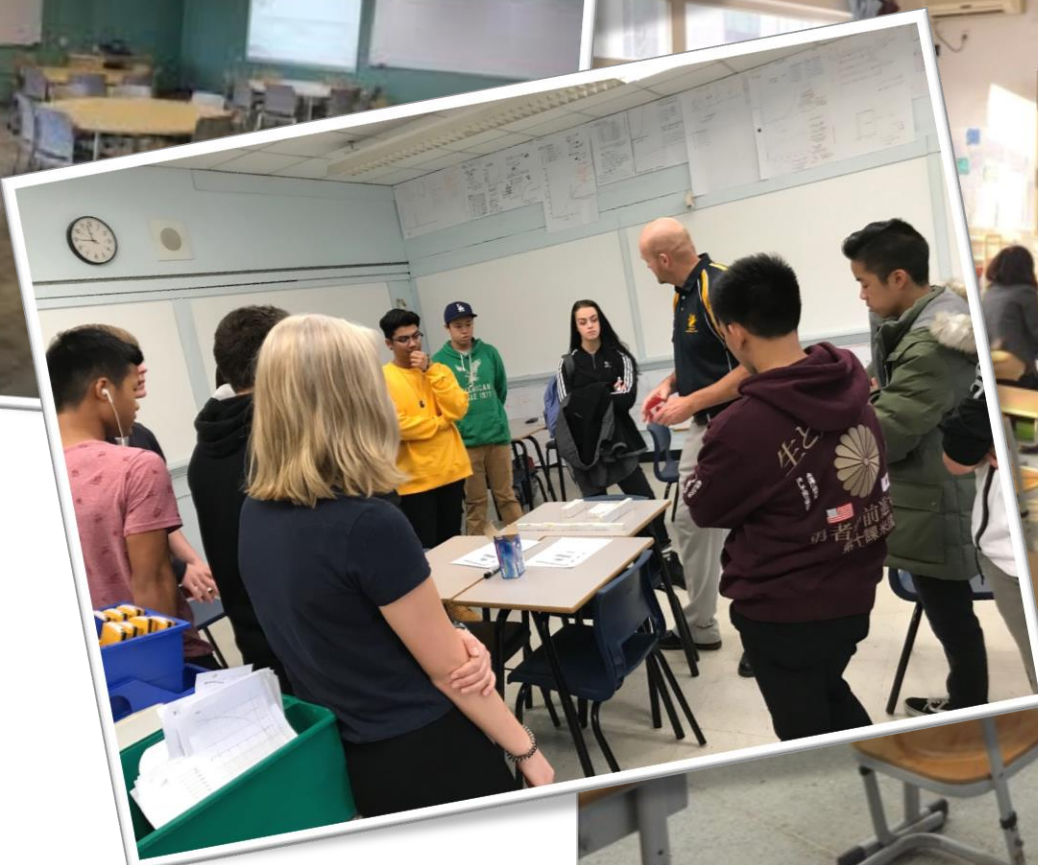
**HOW WE ANSWER
QUESTIONS**



PROXIMITY QUESTIONS
STOP THINKING QUESTIONS
KEEP THINKING QUESTIONS

**HOW WE ANSWER
QUESTIONS**

DEFRONT THE CLASSROOM



HOW WE ARRANGE DESKS

HOW WE

VISIBLY RANDOM GROUPS

in math classrooms

Strategic Grouping's Goals

Educational

- pedagogical
- productivity
- peacefulness



Social

- diversity
- integration
- socialization

Visibly Random Groups

students need to see!

~~teacher assigns~~
students choose

••• 3s
are ideal

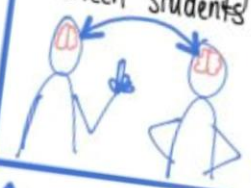
SEPT. 1
Can be introduced ANYTIME in a class

Students become

Eliminates social barriers



Mobility of Knowledge between students



Engagement in task

↑ Enthusiasm for the class (even if the subject is not their favourite)

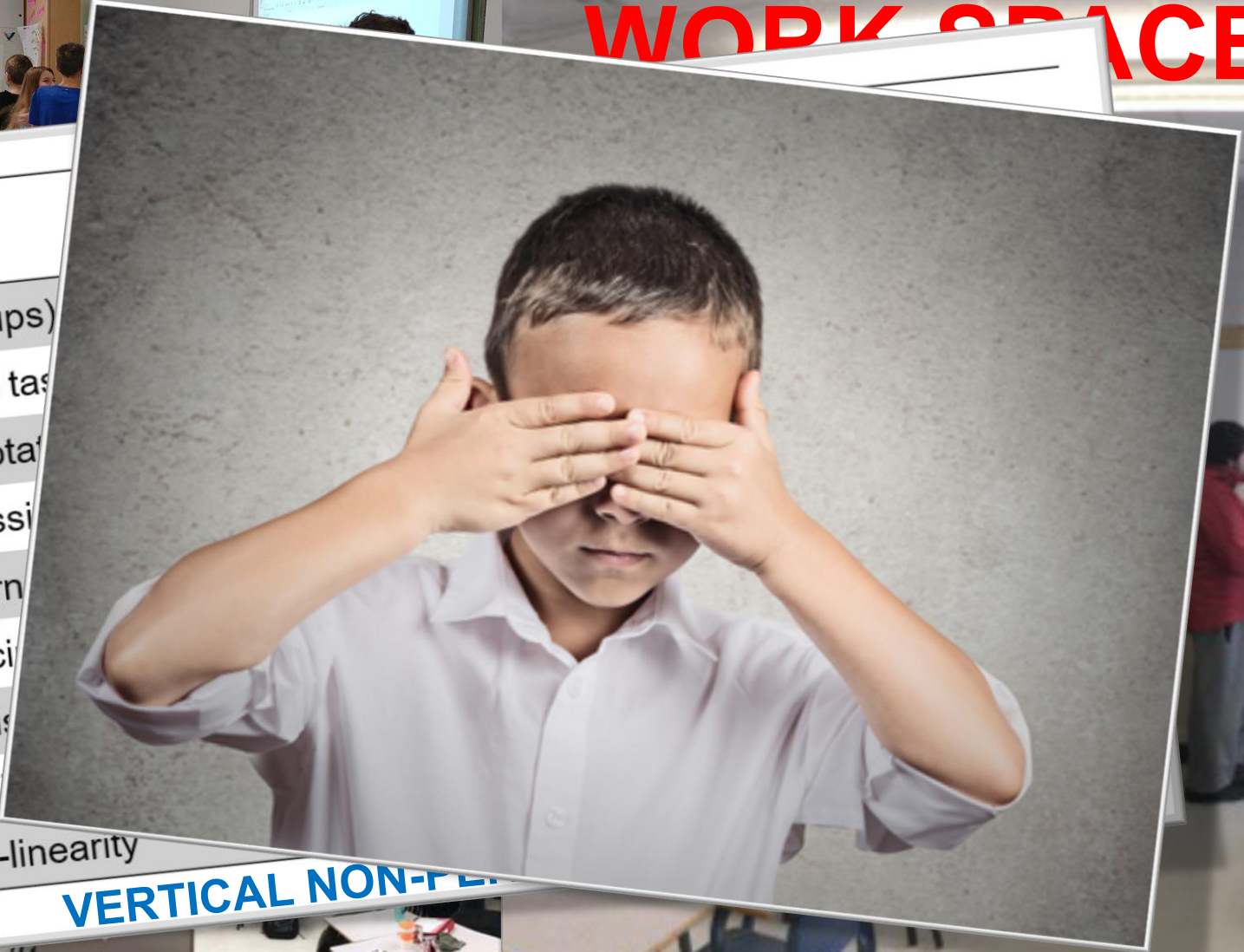


Sketchnote: @wheeler_laura

Research: Peter Lijedahl

- frequently – every 60 minutes
- groups of three are best (except in primary)
- students need to see the randomness

STUDENT WORKSPACE



N (groups)

time to task

first notable

discussion

eagerness

participation

persistence

mobility

non-linearity

VERTICAL NON-FLAT



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BUILDING THINKING CLASSROOMS

RESEARCH: @pgliljedahl
 SKETCHNOTE: @wheeler_laoura

① Begin w/ a Problem

Give a problem-solving task

To start:
 Problems should be
 engaging
 non-curricular
 collaborative
 ↳ promote talking

Later:
 Problems can be curricular
 eg textbook problems

② Visibly Random Groups

- Randomly assigned
eg playing cards
- Daily & in front of students
- 2 or 3 students / group
- Sit & stand together



③ Vertical NonPermanent Surfaces


- Vertical
- Erasable



WHITEBOARD CHALKBOARD WINDOW

- 1 marker or chalk per group
↳ promotes discussion

④ Oral Instructions



give instructions orally

Project

- data
- long expressions
- diagrams


↳ groups will discuss (instead of decoding text)

⑤ Defront the room

Desks

- orient in various directions
- pull away from wall (room to stand @ VNPS)

Teacher addresses the class from a variety of locations.




⑥ Answering Questions

Acknowledge, but don't answer:

- Proximity questions (b/c teacher is close by)
- Stop thinking questions

Answer:
 Keep thinking questions
 ↳ give HINTS not answers



⑦ Meaningful Notes


Student created:

- select
- synthesize
- reorganize

ideas

Based on their or others' boards


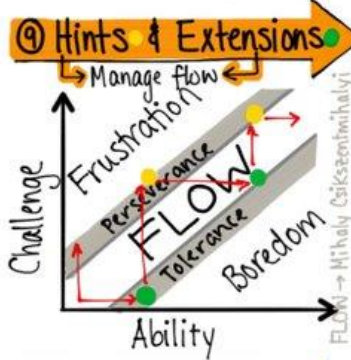
Provide time for this after levelling.



⑧ Build Autonomy

- Model how groups can visit other groups when they are stuck or done.
- Hints & extensions come from peers (not just the teacher).

↳ Helps manage flow

⑩ Level to the Bottom

- debrief
- class discussion
- direct teaching the "lesson"

Once all groups pass a minimum threshold.

- Debrief 1 or more groups' solutions!
- Work through a new problem w/ whole group

⑪ Check Understanding

Assign 4-6 "check for understanding" questions

Students choose to work

- individually
- in groups

at desks on VNPS


Purpose: self-evaluation (NOT marks)

⑫ Formative Assessment

measure → communicate

where student is currently → where student is going

Multiple & varied opportunities to demonstrate learning



can't dis... isn't completely always

fully very

⑬ Summative Assessment

PROCESS > product

Evaluate what you value!

Include:
 group + individual work



⑭ Reporting

Based on data (NOT points)

~~One aggregated mark~~

↳ disaggregated evidence

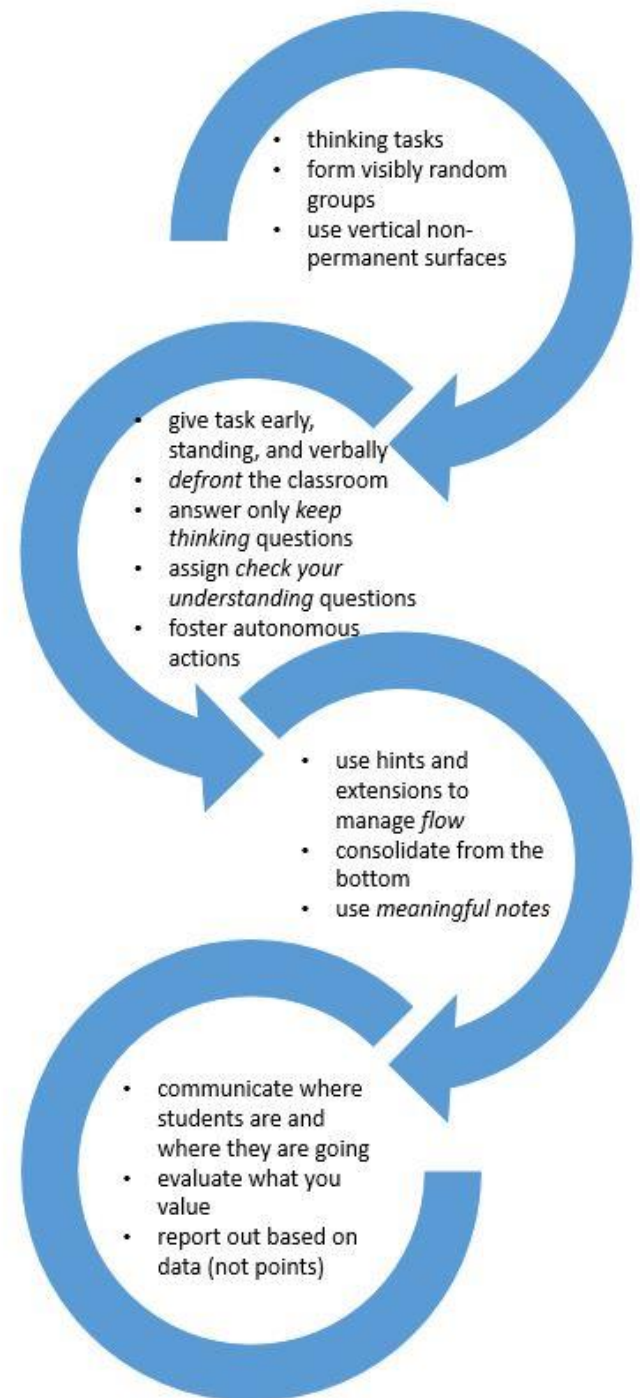
Analysis of data

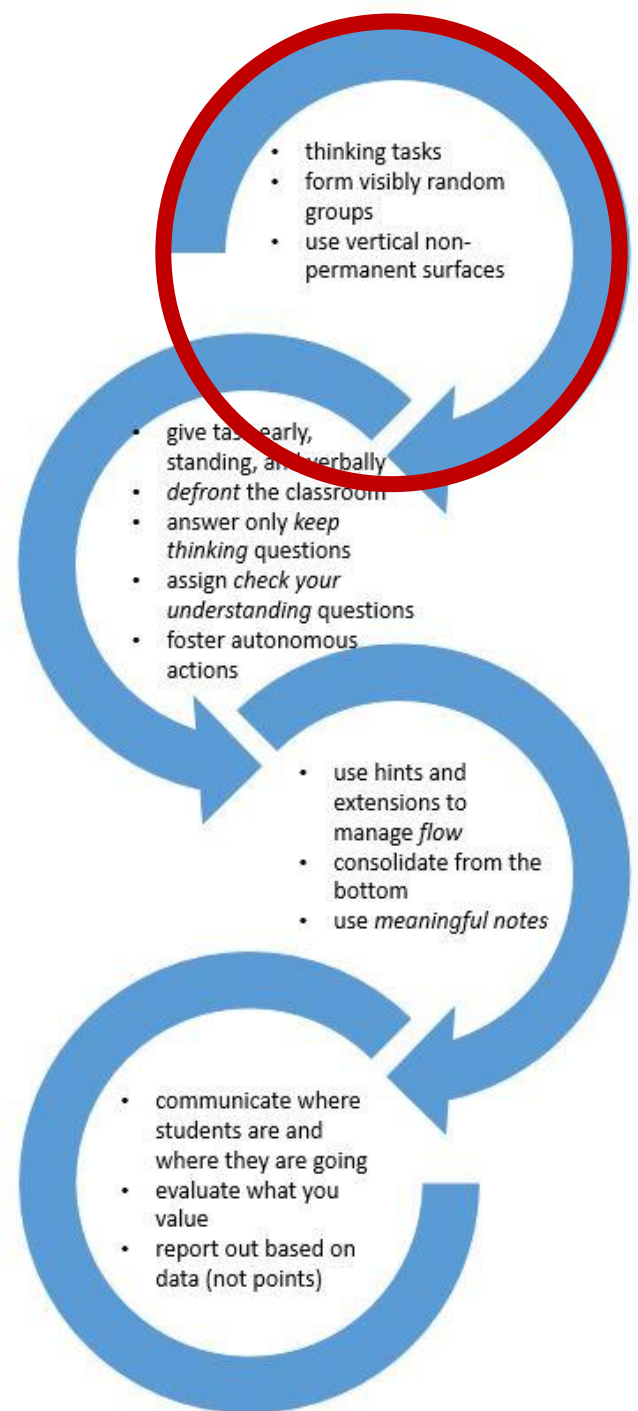
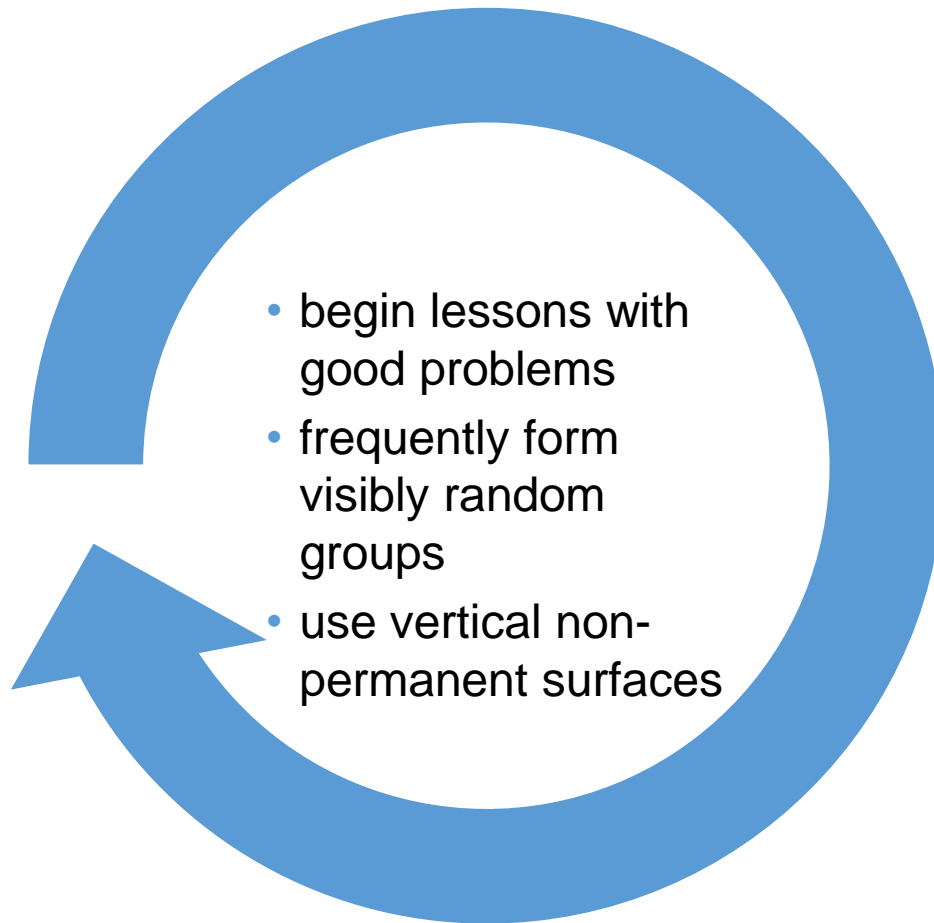
Counting of points

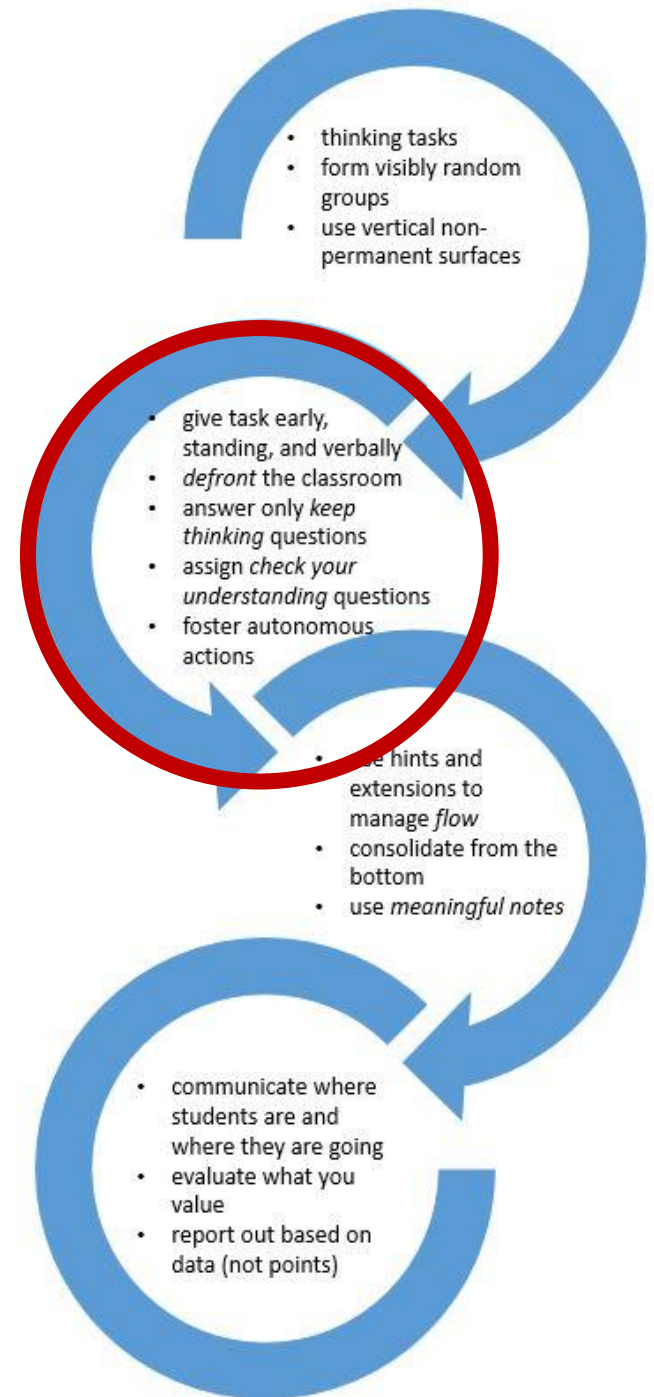
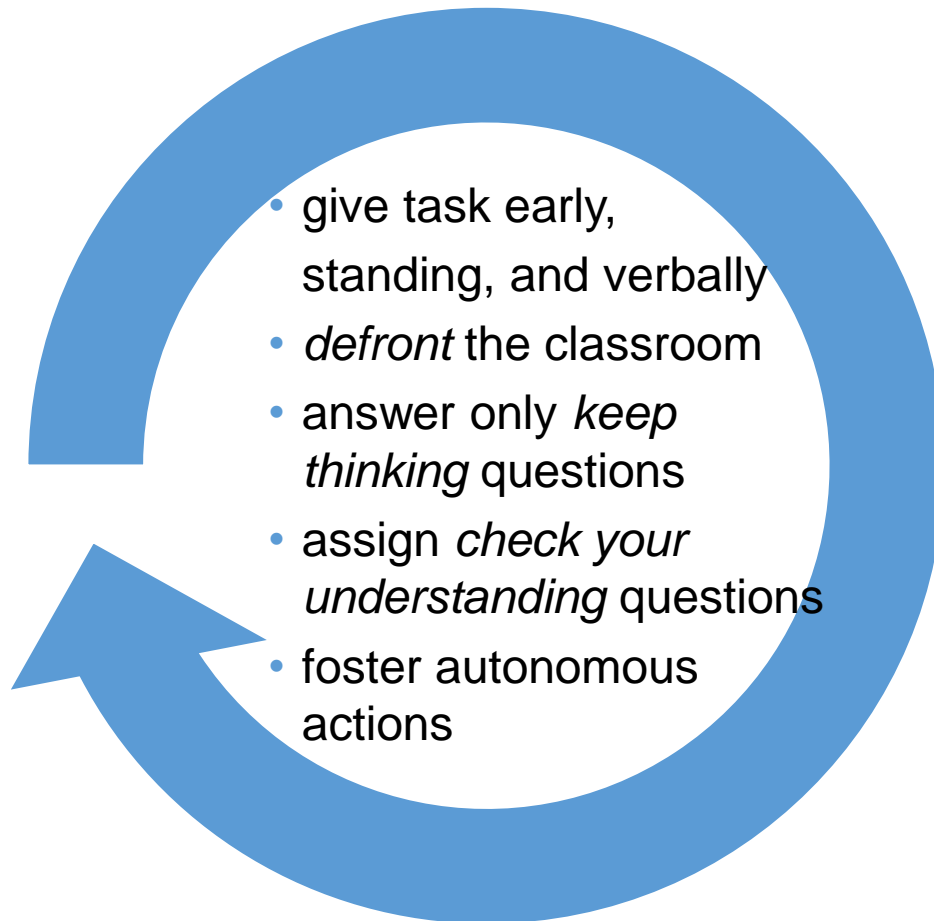
What has this student learned?

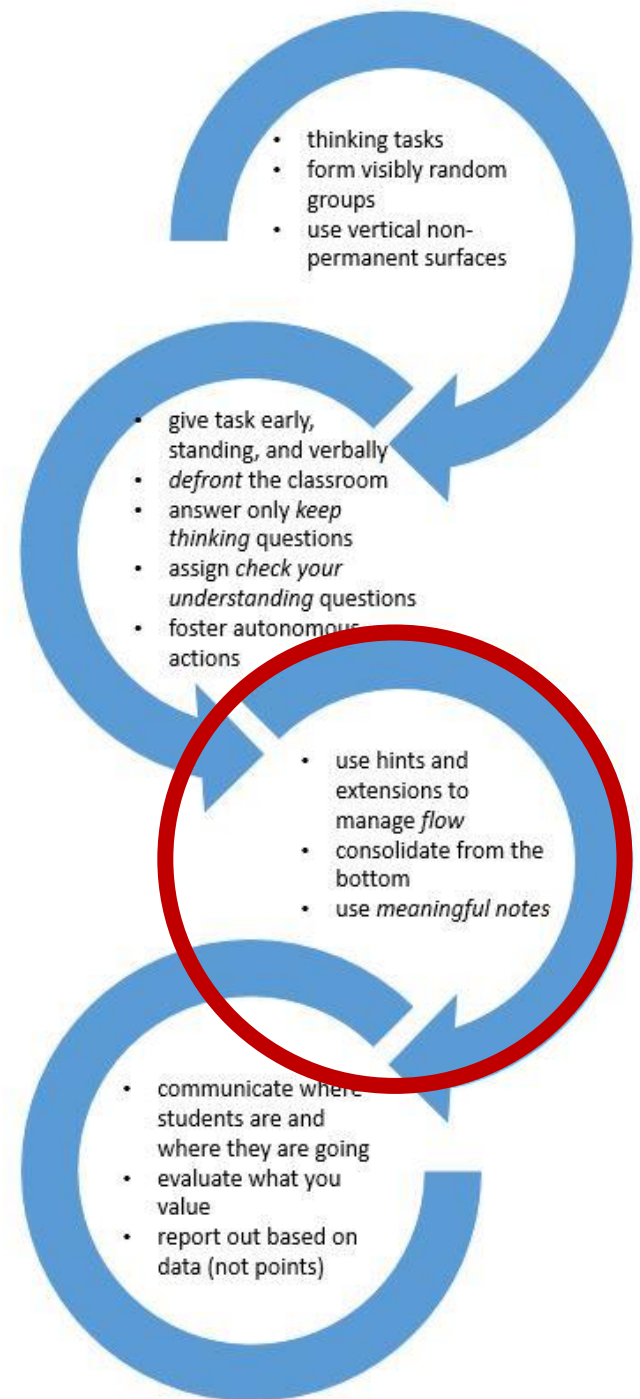
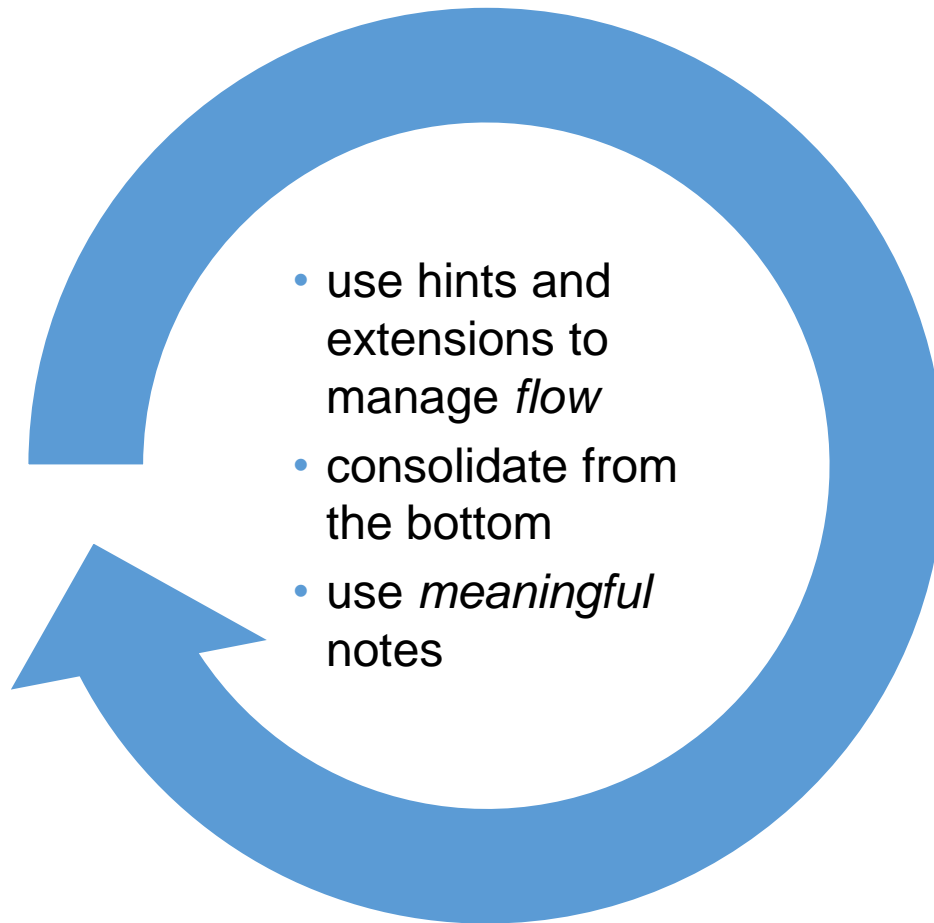
What can they improve?

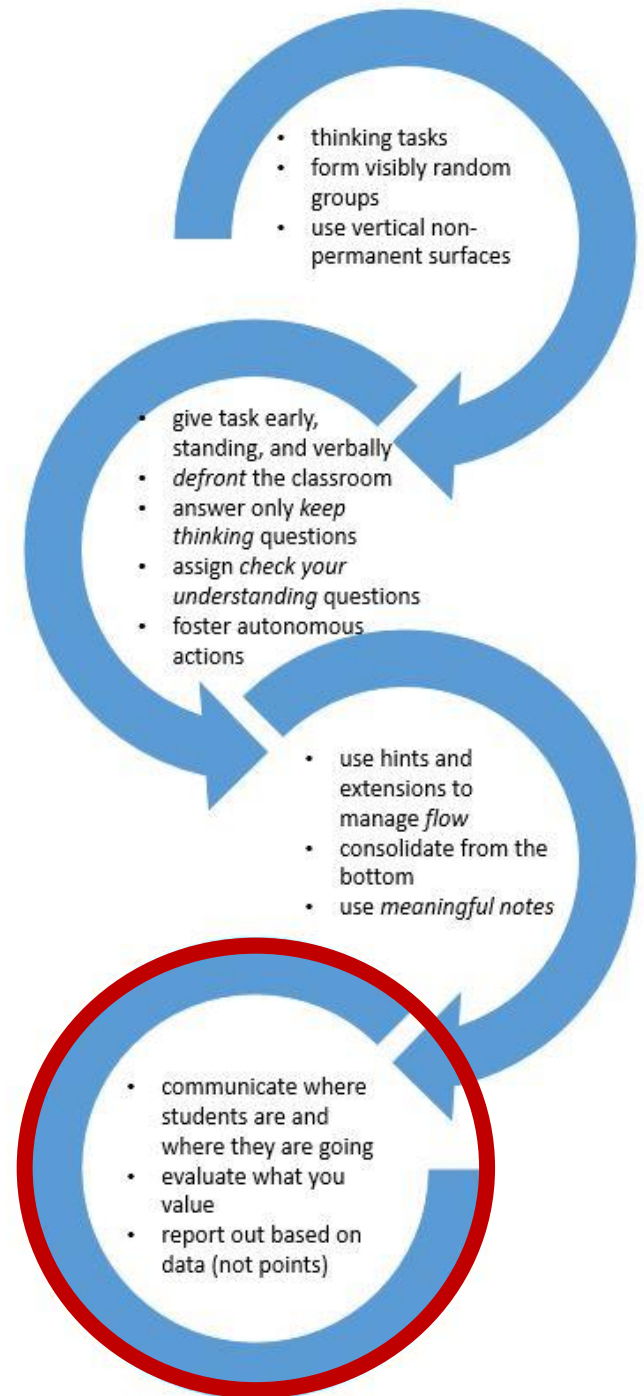
WHERE TO START?



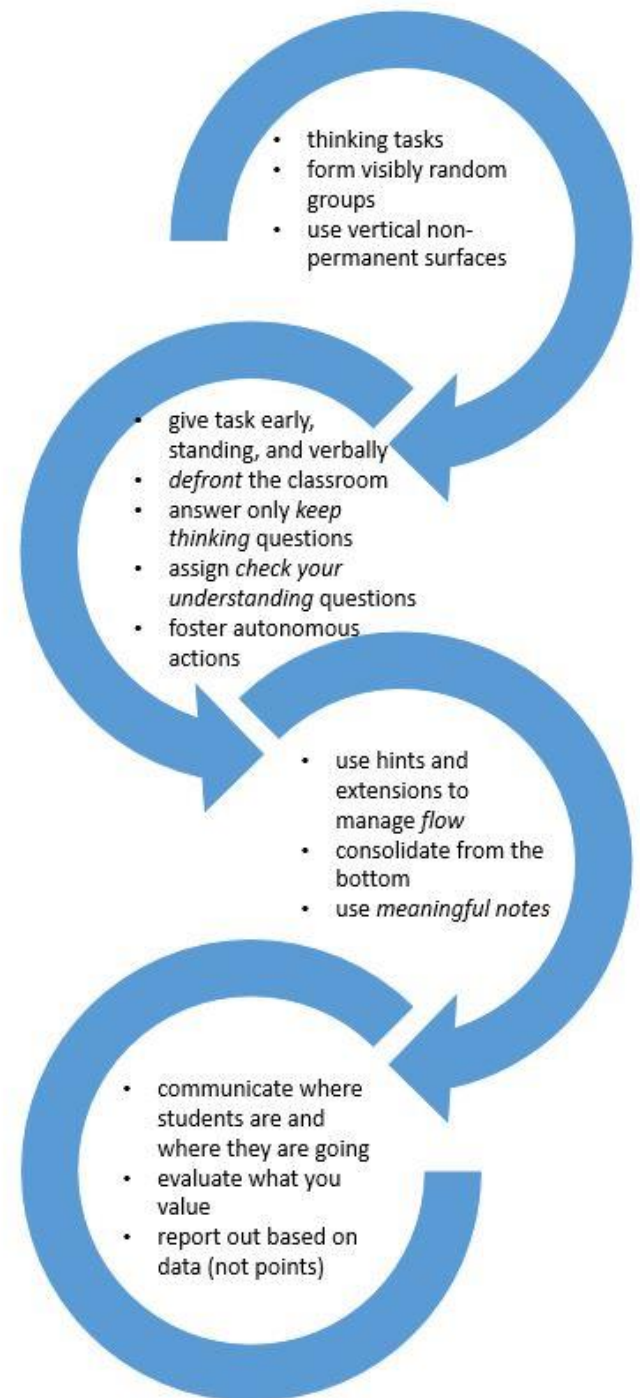








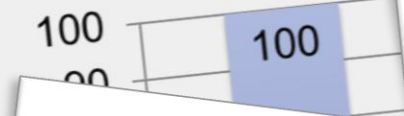
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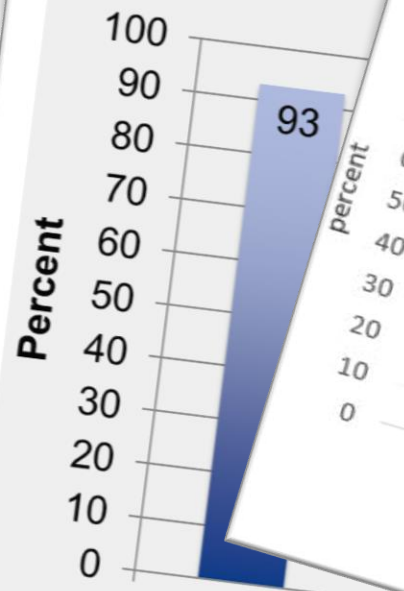
BU
CL

inking tasks
m visibly random

UPTAKE (n=300)



ces



s to keep
it after six
weeks

(n=137)

immediate
intention

tried it in the first
week

still doing it after
six weeks

intends to keep
doing it after six
weeks

where
and
are going
at you

(based on
points)

intends
to try

tries it

after 0
weeks

continue



THANK YOU!



liljedahl@sfu.ca



www.peterliljedahl.com/presentations



@pgliljedahl | #thinkingclassroom



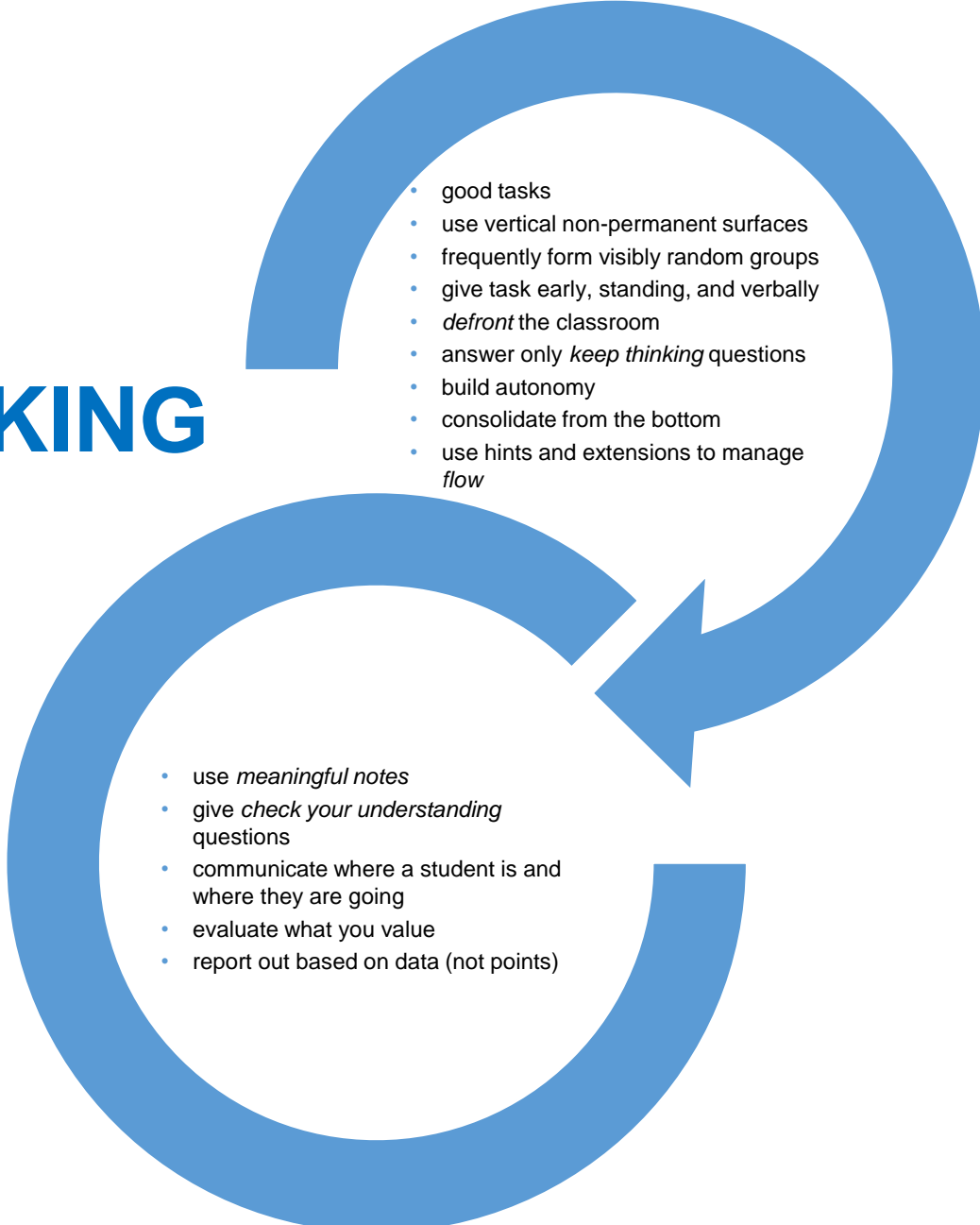
<https://makemathmoments.com/episode21/>



<https://tinyurl.com/y68xopcq>

BUILDING THINKING CLASSROOMS

year II

- 
- good tasks
 - use vertical non-permanent surfaces
 - frequently form visibly random groups
 - give task early, standing, and verbally
 - *defront* the classroom
 - answer only *keep thinking* questions
 - build autonomy
 - consolidate from the bottom
 - use hints and extensions to manage *flow*

- use *meaningful notes*
- give *check your understanding* questions
- communicate where a student is and where they are going
- evaluate what you value
- report out based on data (not points)