

Beyond the Mathematical Competencies: Supporting powerful student learning in mathematics

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The Basic Idea:

I've spent about 20 years studying "powerful mathematics classrooms" – classrooms in which students become knowledgeable and flexible mathematical thinkers as they develop positive mathematical identities.

In the next 45 minutes I'll tell you about about the big ideas – the Teaching for Robust Understanding (TRU) Framework – and introduce two new tools that help create such classrooms.

Outline

- What matters in classrooms? An overview of the ideas in the Teaching for Robust Understanding (TRU) Framework
- Introducing Tool #1, a case study book: “Helping students become powerful mathematical thinkers.” Working through the cases helps us think about our own teaching.
- Introducing Tool #2, “Mathematics Teaching On Target,” a practical guide for enhancing classroom tasks and activities.
- Working in partnership with schools

**The Teaching for Robust Understanding (TRU)
framework, in 3 slides**

(1)

The goal:

To help create classrooms that have the following property:

Every student in this kind of classroom becomes a powerful mathematical thinker who has a sense of mathematical agency and a positive mathematical identity.

(2)

The principal claim concerning TRU:

TRU identifies five key dimensions of mathematics (actually, any) classrooms.

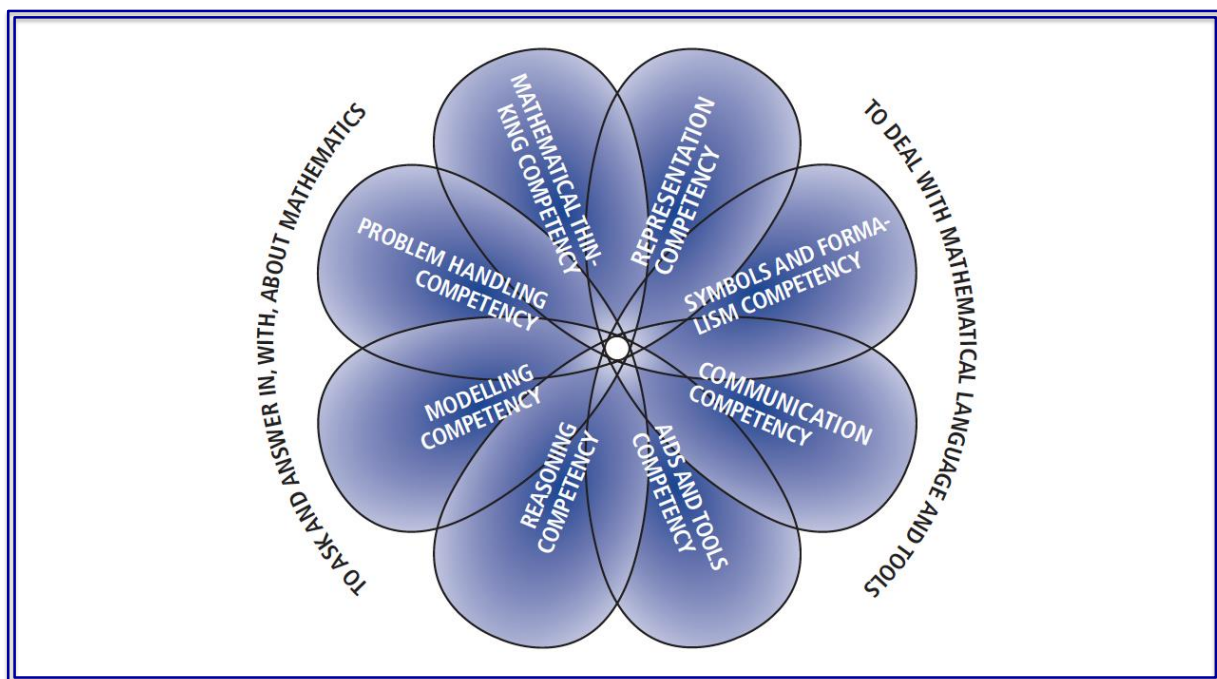
Students will become powerful and agentive mathematical thinkers to the degree that the classroom does well along these 5 dimensions:

(3) The Five Dimensions of Powerful (Mathematics) Classrooms

The Content	Cognitive Demand	Equitable Access to Content	Agency, Ownership, and Identity	Formative Assessment
<p><i>The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful disciplinary thinkers. Discussions are focused and coherent, providing opportunities to learn disciplinary ideas, techniques, and perspectives, make connections, and develop productive disciplinary habits of mind.</i></p>	<p><i>The extent to which students have opportunities to grapple with and make sense of important disciplinary ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called "productive struggle."</i></p>	<p><i>The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core disciplinary content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.</i></p>	<p><i>The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations about disciplinary ideas, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.</i></p>	<p><i>The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.</i></p>

(Footnote to slide 3...)

Now, it's not as though I'm the first person to talk about mathematical content...



You can find tons of detail about
TRU at
<https://truframework.org/>

What's important about this framework?

Here are a few central points.

1. The TRU Dimensions are necessary and sufficient. That is,

If things go well along all 5 dimensions, students will emerge from the classroom being powerful thinkers.

If things go badly along *any* of the dimensions, they will not.

So, the key idea is to get better at all 5 dimensions.

2. TRU involves a fundamental shift in perspective, from teacher-centered to student-centered.

The key question is *not*: “Do I like what the teacher is doing?” It is:

“What does instruction feel like, from the point of view of the student?”

Note that four of the five TRU dimensions are about the student’s experience of mathematics.

Observe the Lesson Through a Student’s Eyes

The Content

- What’s the big idea in this lesson?
- How does it connect to what I already know?

Cognitive Demand

- How long am I given to think, and to make sense of things?
- What happens when I get stuck?
- Am I invited to explain things, or just give answers?

Equitable Access to Content

- Do I get to participate in meaningful math learning?
- Can I hide or be ignored? In what ways am I kept engaged?

Agency, Ownership, and Identity

- What opportunities do I have to explain my ideas? In what ways are they built on?
- How am I recognized as being capable and able to contribute?

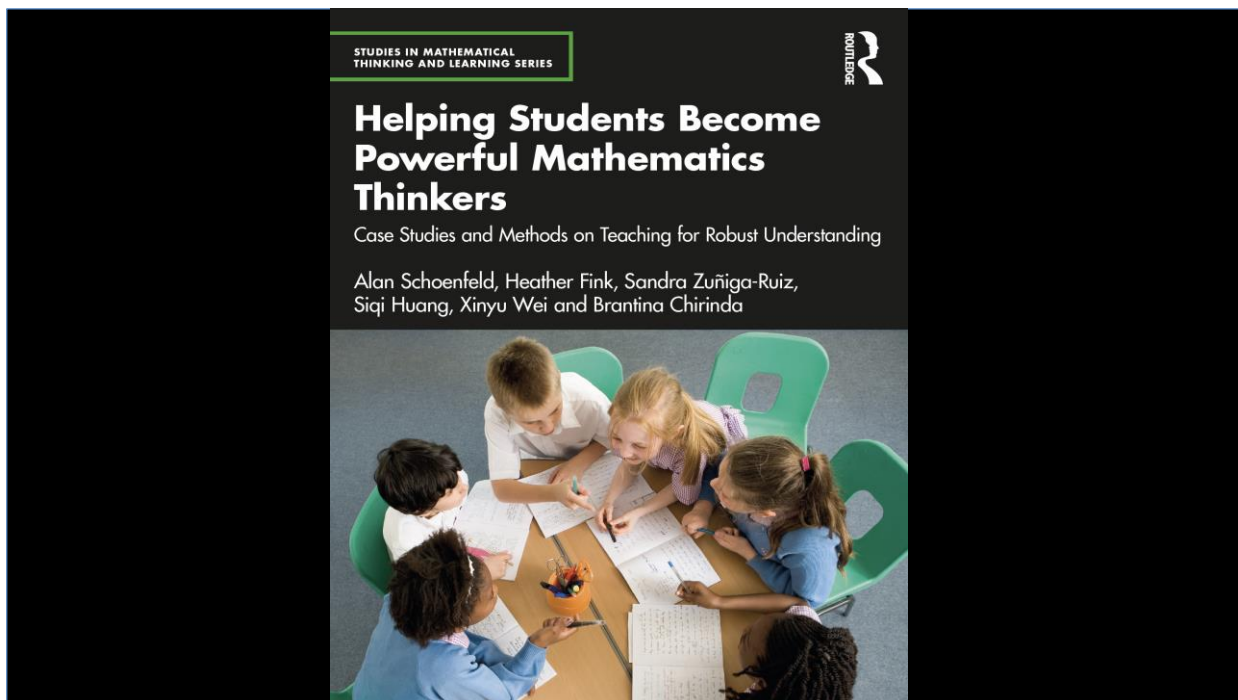
Formative Assessment

- How is my thinking included in classroom discussions?
- Does instruction respond to my ideas and help me think more deeply?

3. TRU does not tell you how to teach, because there are many different ways to be an effective teacher.

TRU describes the *principles* of powerful instruction, so it serves to *problematize* instruction. Asking, “how am I doing along this dimension; how can I improve?” makes your teaching richer, without telling you what to do. Today’s presentation introduces two new tools to help you do that.

Let me turn to our first new tool,
a book of case studies of teaching.



In the book we look *very* closely at a number of classrooms, exploring possibilities.

I'm going to walk you through one of the cases, to give you a sense of what we do.

Imagine the idea of a “post-game re-play.”
When something interesting happens, we
stop to consider:

What *might* a teacher do, what might the
consequences be?

The idea is NOT to judge the teacher, but to
think about the possibilities. How can we think
about the impact of different decisions on the
students?

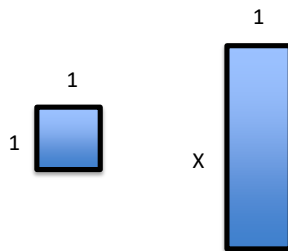
Where is the 10?

We are joining a 9th grade “Sheltered Algebra”
class. The students are native Spanish speakers,
with varied degrees of fluency in English (from 0
to fluent.)

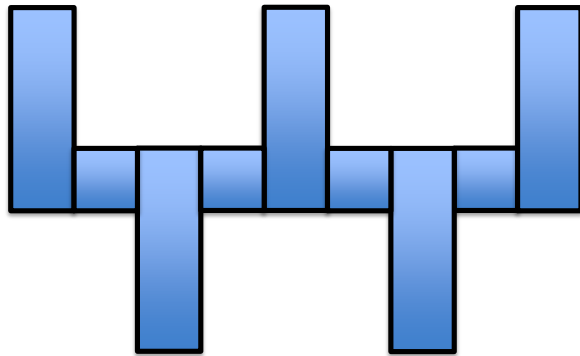
This is a “Complex Instruction” class.
Some features of instruction:

1. The problems are very rich mathematically, with opportunities for engaging with valuable mathematical content and practices.
2. Students are responsible for helping each other. For each task one student is the “explainer,” and the others help that person understand and explain.

The class is working on problems using these algebra tiles:



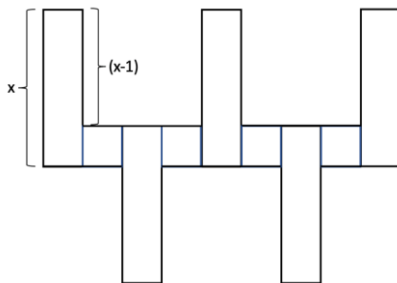
They're given this figure. What is its perimeter?



If we were working together for a few hours, we'd all work the problem – in as many ways as possible.

Here I'll show you a few solutions, and then ask a more difficult question.

Method 1

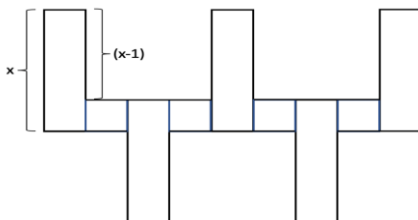


The "inside" parts of the rectangles

Going all the way around the perimeter starting at the left, you get
 $x + 1 + (x - 1) + 3 + (x - 1) + 1 + (x - 1) + 3 + (x - 1) + 1 + x + 2 + (x - 1) + 1 + (x - 1) = 10x + 10.$

Method 2

Too much work! Algebra's helpful:



Two long sides of length x : $x + x = 2x$

Eight medium sides of length $(x-1)$: $8(x - 1) = 8x - 8$

Eighteen short sides of length 1: 18

Add the sorted lengths together to find the total perimeter,

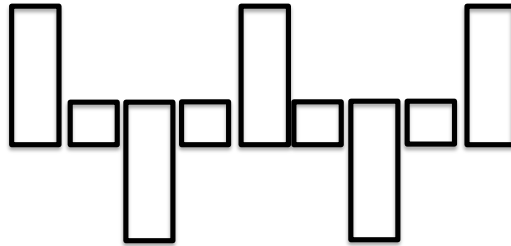
$$2x + 8x - 8 + 18,$$

and combine like terms to get the final answer,

$$10x + 10.$$

Method 3

Take the figure apart.

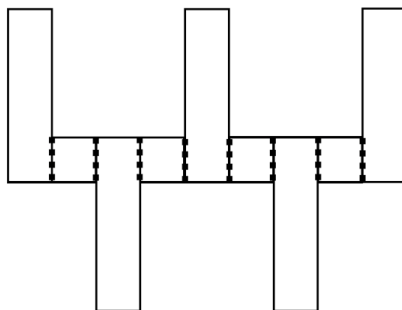


The five rectangular tiles each have perimeter of $2x + 2$, and the four square tiles each have a perimeter of 4. Adding the perimeters of the tiles together, you get:

$$\begin{aligned} 5(2x + 2) + 4(4) \\ = 10x + 26. \end{aligned}$$

BUT...

But don't forget about the overlaps! The tiles overlap in eight places. Each overlap is one unit long.



You have to subtract the overlapping lengths *twice*, since every overlap results in losing a length of 1 from *both* overlapping tiles. Subtract 16 and you get

$$\begin{aligned} 10x + 26 - 16 \\ = 10x + 10. \end{aligned}$$

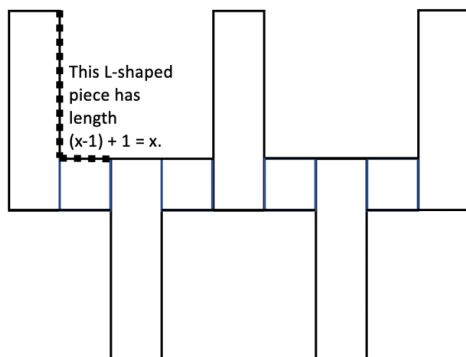
OK, the answer is “ $10x + 10$ ”.
That’s a nice lesson intro; you can imagine
comparing solutions.

But the next question is harder:
“Show me the 10 in the figure.”

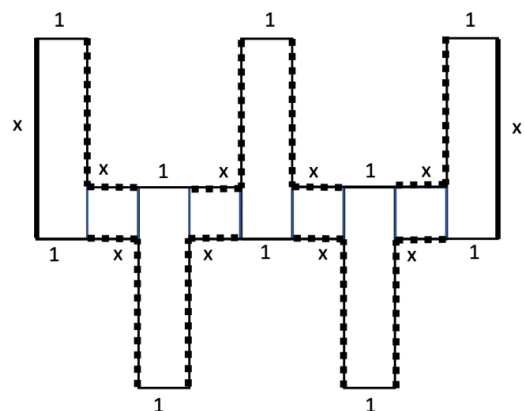
You might be able to use one of the
methods we used before, but showing
exactly why there are 10 sections of length 1
in the perimeter is not easy!

The students in the classroom we visit find it
very hard.

Here are two approaches my students and their students have come up with...

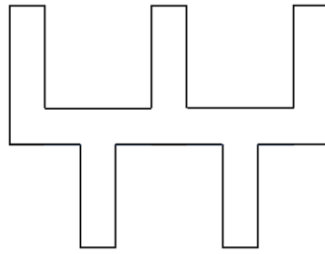


The L-shaped piece has length x .

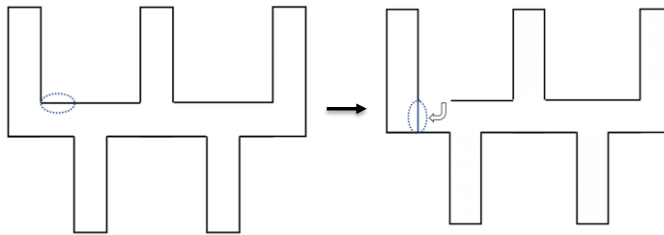


With 8 L-shaped pieces there are 10 pieces of length x and 10 pieces of length 1.

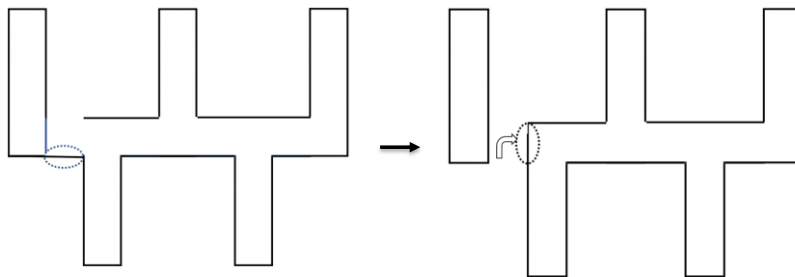
Start with this:



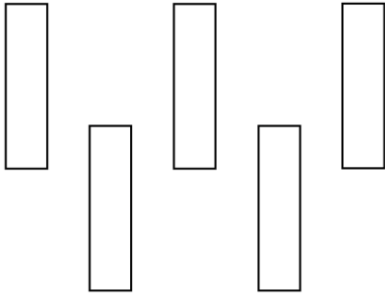
“Move” the circled segment like this:



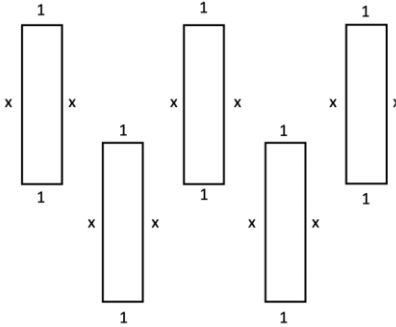
Then move the next segment as indicated...



And keep going until you get this:



Which, with the sides labeled, is



It's pretty easy to see both the $10x$ and the 10 , as desired.

There are other ways to get $10x + 10$, but we'll stop here.

Why do all this? Here's a question.

What mathematical understandings might students develop from engaging in this lesson? How are those understandings important?

Here are some possible responses.

We think that the original on-paper task and the teacher's follow-up question have the potential to help beginning algebra students:

- Make connections between algebraic and geometric representations,
- Learn how to combine like terms and do algebraic manipulation by applying new algebraic knowledge in a geometric context, and,
- Explain their thinking about algebra, both as they're engaged in sense making and in summary, to their classmates and peers.

How can challenging tasks like this one support rich mathematical learning environments that help build every student's agency, ownership, and identity in math?

We consider this task to hold potential for supporting the development of students' agency, ownership, and identity (Dimension 4) – and more! – because:

- It provides students with opportunities to connect prior knowledge with new, essential, and interesting concepts. Doing some of this work may be a challenge – we'll see it as the case unfolds – but the level of challenge can be kept within reach. That's a matter for formative assessment (Dimension 5) and cognitive demand (Dimension 2).
- There are multiple ways to reach a solution. This contributes to the students' developing sense of what mathematics is, and what it means to do mathematics (Dimension 1) and provides multiple points of access (Dimension 3).
- Similarly with regard to access, there are various ways for students to contribute meaningfully to the solving process.

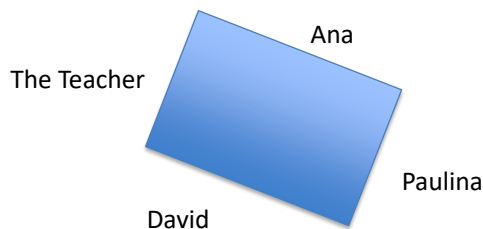
This is what happened.

We visit a group of students in the middle of the lesson.

The teacher has asked the students, "Show me the ten".

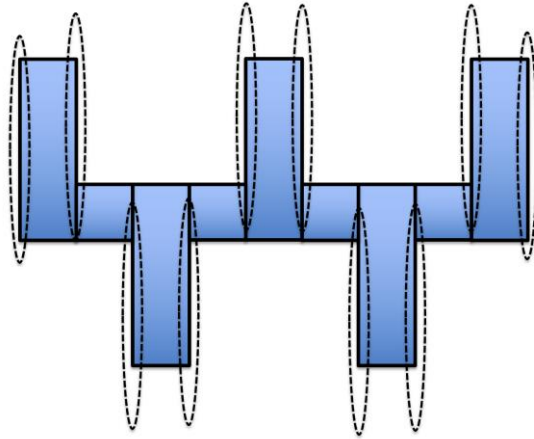
This part of the case study begins when the students tell the teacher that they have the answer.

Ana is the designated speaker for the group.



What happened (part 1)

Ana starts to count, but she counts the X's, not the 1's:



What happened, continued...

The teacher says, "I asked you for the ten. You're counting the X's." When it becomes clear that Ana can't produce the answer. On her own, the teacher says, "You need to work together. I'll be back." The students continue working.

David says to Ana, "Just tell her this..." but Ana cuts him off, saying "You know she's going to want a thorough explanation. You have to explain it to me so that I really understand it." It's clear from their exchanges that David and Paulina consider Ana's math to be weak.

They keep working until they all think they all understand. They call the teacher.

Once again it becomes clear that they don't really understand, and the teacher says that she'll be back.

Here are some questions to consider.

(We ask a lot more in the book. There's more time to work through them!)

How do the interactions in these two episodes appear to position each of the students with regard to their mathematical proficiency? Although it's very early in the vignette and we want to be careful about drawing conclusions, are there points to note about the interactions that might play out with regard to AOI?

As they work the students switch back and forth between Spanish and English. What role does the use of both languages play? How does it appear to affect opportunities for mathematical sense making? Moreover, we're privileged to see some things the teacher doesn't, in terms of the ways Alicia, Bernardo and Carla interact. There's some banter. Is it just "time off task?" Does it play a role in any potentially productive ways?

As the lesson continues...

The students find the problem really hard. They think they have a solution, but it falls apart. Ana gets very frustrated, but she insists that David and Paulina continue working with her – and she gets it.

Finally, she is able to explain to the teacher where you can see the 10 in the figure. When she works on the next problem, you can see that she is much more confident.

Here are sample questions we ask, as the lesson progresses:

What can you say about emerging mathematical understandings?
What happens here, with regard to both mathematical content and practices?

Things are a bit tense as the students are positioned (either tacitly or explicitly) by each other when they interact with Ms. Davis. Is there anything to note about what the students say, or about possible implications of how Ms. Davis deals with the situation?

What can you say about AOI? What are the tensions, what do you want to keep your eyes on?

Some more questions to consider...

Once again, let's start with the mathematics. What was the group's final explanation for why there are 10 1's in the perimeter, and 10 x's? Is it fully correct? Might there be more to say?

Things were tense at times in the first three episodes. The three students were obviously comfortable with each other, but at times David and Paulina were somewhat dismissive of Ana... to the point where one might be tempted to intervene as a teacher. How did things work out? What can we say about each student's participation, and possibly their mathematical senses of self?

Any other issues you want to raise?

And now a sense of our discussion questions...

There's so much more to say!

Dimension 1, the Mathematics

The Mathematics

- What are the big ideas in this lesson?
- How do they connect to what I already know?



As we saw in Section 2.1, there's the potential for a lot of rich mathematics to emerge from discussions of the algebra tiles task at the heart of this case study. What do you think about the mathematics that emerged in the conversations between Alicia, Bernardo, Carla, and Ms. Davis? In what ways was it challenging and rich? How might the choice of mathematical topic and task design have shaped what the students experienced? What kinds of mathematical expectations were established for and by the students, and how did they play out?

Please think about and discuss these issues before you read our commentary.

Dimension 2, Cognitive Demand

Cognitive Demand

- How long am I given to think, and to make sense of things?
- What happens when I get stuck?
- Am I invited to explain things, or just give answers?



The idea behind cognitive demand is that students learn best from “productive struggle” – that it's good for them to be stretched, but not to the breaking point. What individual and/or collective challenges did Alicia, Carla, and Bernardo encounter? What resources were available to each of them to meet those challenges?

Please think about and discuss these issues before you read our commentary.

Dimension, 3 Equitable Access

Equitable Access to Content

- Do I get to participate in meaningful math learning?
- Can I hide or be ignored? In what ways am I kept engaged?



Equitable Access is “the extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematical content and practices.” To what degree were each of the three focal students in this case study invited to engage in the core content and practices described in Section 2.5.1, what opportunities did they have to participate, and in what ways were they each supported in them?

Please think about and discuss these questions before you read our thoughts.

Dimension 4, Agency, Ownership, and Identity

Agency, Ownership, and Identity

- What opportunities do I have to explain my ideas? In what ways are they built on?
- How am I recognized as being capable and able to contribute?



Issues of agency concern students’ willingness to dig into mathematical challenges. **Ownership** relates to students making the math their own, rather than implementing procedures they’re given. **Identity** concerns how the students see themselves as doers of mathematics. These are all shaped by the classroom interactions – how students are positioned, what they’re responsible for, which resources (including linguistic resources) they feel comfortable bringing to bear, and the kinds of progress they make. What evidence do we have about these issues, for each of the three students?

Please think about and discuss these questions before you read our thoughts.

Dimension 5, Formative Assessment

Formative Assessment

- How is my thinking included in classroom discussions?
- Does instruction respond to my ideas and help me think more deeply?

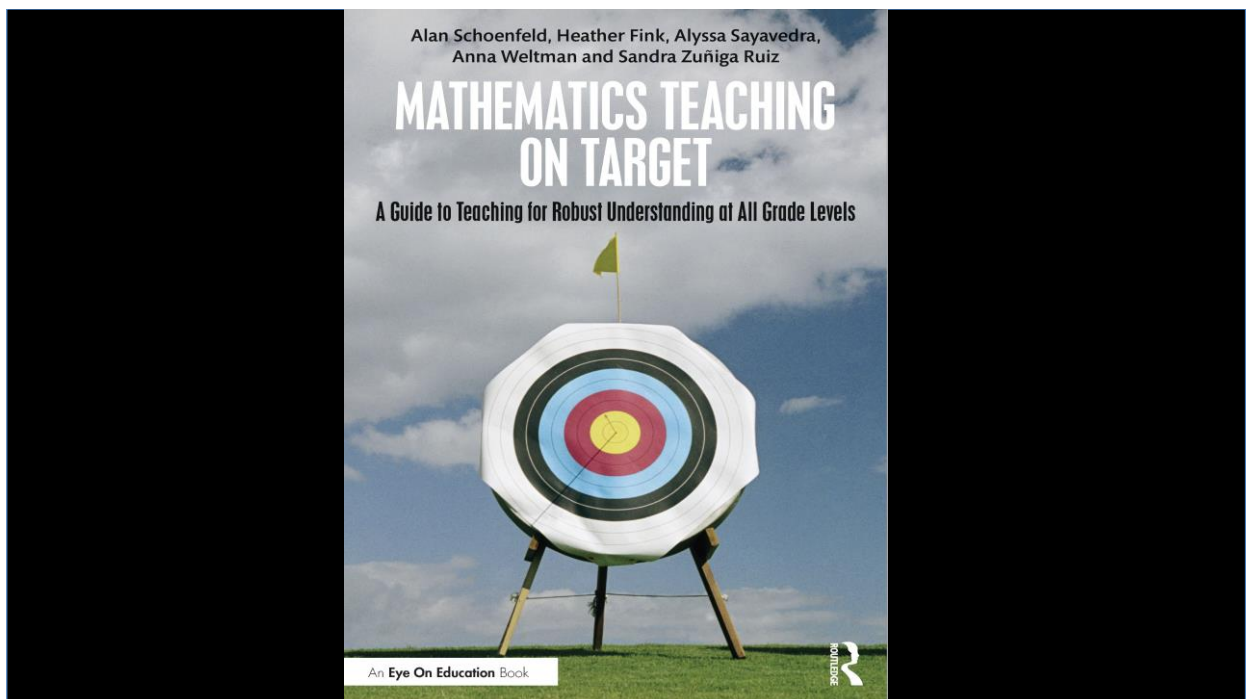


Formative Assessment involves making students' thinking public and, if necessary, adjusting classroom activities to support students in engaging meaningfully and productively with the content. The resources available are not only the curriculum and the teacher, but other students as well. In what ways did formative assessment play out in this case study, with regard to rich mathematics and the opportunities each of the three students had to engage meaningfully with it?

Please think about and discuss these questions before you read our thoughts.

I hope you can imagine spending a significant amount of time working through issues like these, reflecting on them, and applying the ideas to your own teaching. The book offers lots of opportunities.

**Here's our second new tool, for enriching tasks
and activities:**



The big question is,

What can we do to make tasks or activities mathematically richer, in ways that engage students more and enrich all 5 dimensions?

How can we:

- Enrich the math?
- Provide more opportunities for productive struggle?
- Make tasks or activities more accessible?
- Open up tasks or activities for meaningful positive engagement?
- Design for productive feedback?

The basic metaphor is a target.
Consider an activity.

Is it decent but not exciting?

Is it good but could it be better?

Is it really "on target"?

How can you move it more to the center?

I'll use the math dimension to show
you how *On Target* works.
At the end I'll quickly show you the other 4
dimensions.

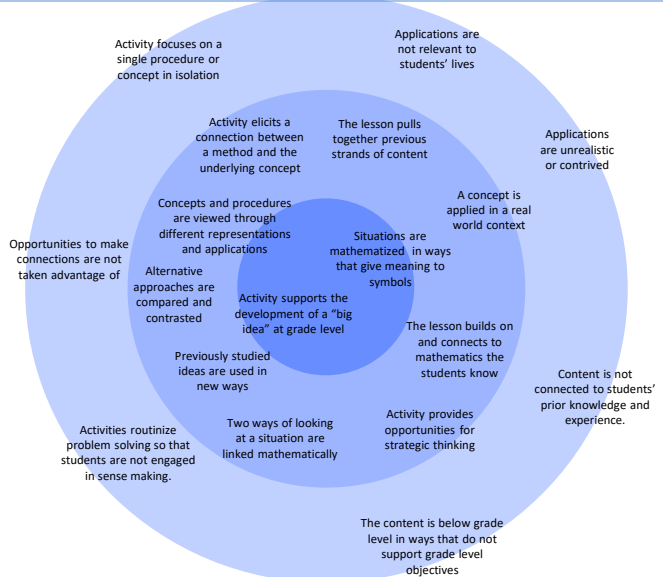
**The
Mathematics**

In what ways do classroom activities provide opportunities for students to become knowledgeable, flexible, and resourceful mathematical thinkers?

Core Questions:

- 1. What is the main mathematical idea?** How does it develop?
How is it connected to what students know? How is it connected to the grade level content and practice standards?
- 2. In what ways do students engage with the mathematical content?** What connections are built between procedures, underlying concepts, and meaningful contexts of application?
- 3. In what ways do students engage in mathematical practices** or other activities that build productive mathematical habits of mind?

The Mathematics **1. What is the main mathematical idea? How does it develop? How is it connected to what students know? How is it connected to the grade level content and practice standards?**



The Mathematics **2. In what ways do students engage with the mathematical content? What connections are built between procedures, underlying concepts, and meaningful contexts of application?**



**The
Mathematics**

3. In what ways do students engage in mathematical practices or other activities that build productive mathematical habits of mind?



We'll take 3 typical tasks
(1 elementary, 1 middle, 1 secondary)
and think together about how to
improve them.

1. An Elementary grades example: Two-Digit Subtraction

Imagine you're at the point in the curriculum where your students have learned two-digit subtraction. Here's a typical sheet of practice examples using "borrowing."

$$\begin{array}{r} 1) 42 \\ - 18 \\ \hline \end{array}$$

$$\begin{array}{r} 2) 62 \\ - 35 \\ \hline \end{array}$$

$$\begin{array}{r} 3) 55 \\ - 28 \\ \hline \end{array}$$

$$\begin{array}{r} 4) 71 \\ - 49 \\ \hline \end{array}$$

$$\begin{array}{r} 5) 70 \\ - 37 \\ \hline \end{array}$$

$$\begin{array}{r} 6) 42 \\ - 18 \\ \hline \end{array}$$

$$\begin{array}{r} 7) 73 \\ - 9 \\ \hline \end{array}$$

$$\begin{array}{r} 8) 36 \\ - 19 \\ \hline \end{array}$$

$$\begin{array}{r} 9) 54 \\ - 15 \\ \hline \end{array}$$

$$\begin{array}{r} 10) 81 \\ - 23 \\ \hline \end{array}$$

$$\begin{array}{r} 11) 68 \\ - 39 \\ \hline \end{array}$$

$$\begin{array}{r} 12) 43 \\ - 17 \\ \hline \end{array}$$

$$\begin{array}{r} 13) 84 \\ - 26 \\ \hline \end{array}$$

$$\begin{array}{r} 14) 92 \\ - 27 \\ \hline \end{array}$$

$$\begin{array}{r} 15) 80 \\ - 22 \\ \hline \end{array}$$

$$\begin{array}{r} 16) 53 \\ - 16 \\ \hline \end{array}$$

$$\begin{array}{r} 17) 77 \\ - 38 \\ \hline \end{array}$$

$$\begin{array}{r} 18) 64 \\ - 35 \\ \hline \end{array}$$

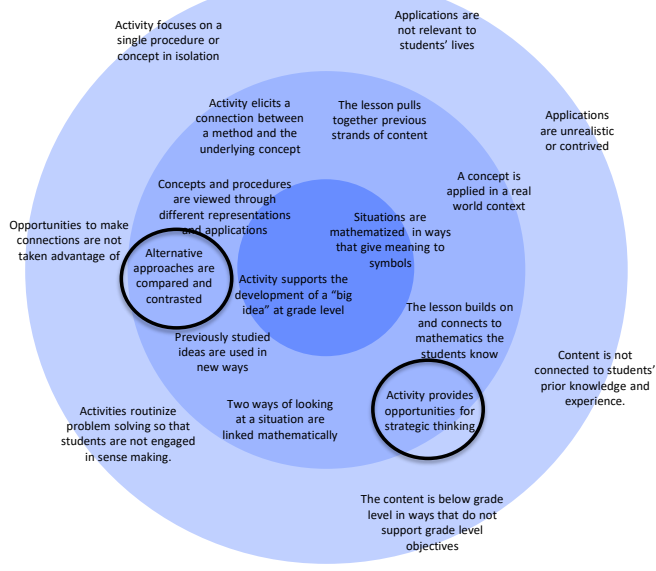
$$\begin{array}{r} 19) 74 \\ - 58 \\ \hline \end{array}$$

$$\begin{array}{r} 20) 62 \\ - 36 \\ \hline \end{array}$$

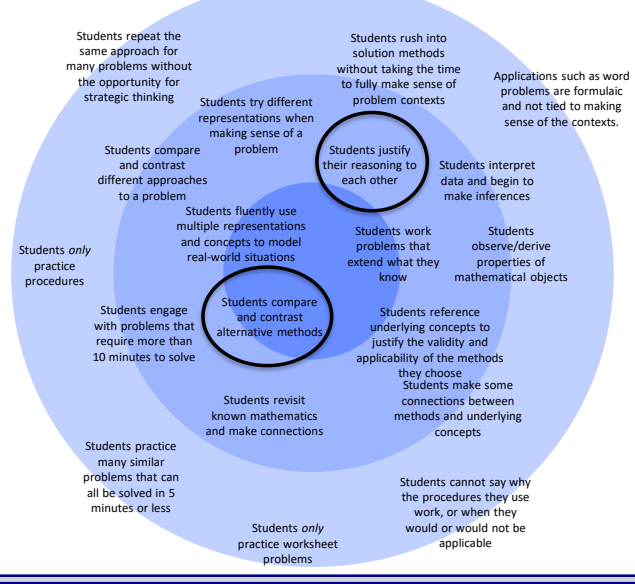
Practicing skills is useful. But are there ways we can modify or add to the practice sheet so that students get more out of working it?

Let's explore the 3 targets.

The Mathematics **1. What is the main mathematical idea? How does it develop? How is it connected to what students know? How is it connected to the grade level content and practice standards?**

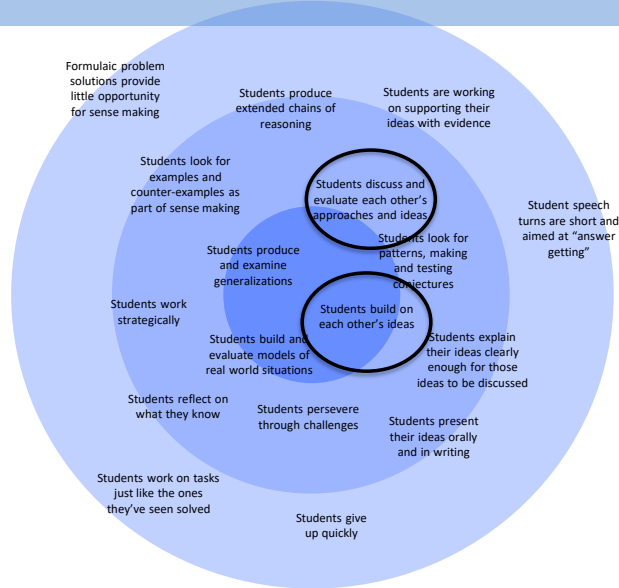


The Mathematics **2. In what ways do students engage with the mathematical content? What connections are built between procedures, underlying concepts, and meaningful contexts of application?**



The Mathematics

3. In what ways do students engage in mathematical practices or other activities that build productive mathematical habits of mind?



In sum, here are the possibilities we flagged.

On Target 1, we focused on these opportunities for enrichment:

- Alternative approaches are compared and contrasted
- Activity provides opportunities for strategic thinking

On Target 2, we focused on these:

- Students justify their reasoning to each other
- Students compare and contrast alternative methods

And on Target 3:

- Students work on supporting their ideas with evidence
- Students discuss, evaluate, and build on each other's ideas

Any ideas about how to modify the task?

Some thoughts:

- All these tasks use “borrowing.” What about mixing things up, including some subtractions like

$$\begin{array}{r} 87 \\ - 53 \\ \hline \end{array} ?$$

- When discussing the first task,

$$\begin{array}{r} 43 \\ - 19 \\ \hline \end{array} ,$$

ask if anyone has solved the problem a different way.

Some students may say that they changed the problem to

$$\begin{array}{r} 44 \\ - 20 \\ \hline \end{array} ,$$

which they thought was easier to do. You can run with this:

- Does it always work, when do you want to use it, etc?

- Does it make sense to use this method for

$$\begin{array}{r} 79 \\ - 18 \\ \hline \end{array} ,$$

for example?

You can also do this as a “number talk...”

2. A middle grades example. - Graphing Lines

Imagine that you’re at the point in the middle grades curriculum where your students have studied different forms that represent the graphs of straight lines

– e.g.,

The point-slope formula; standard form; slope-intercept form; two-point form; two-intercept form.

Typical exercises call for sketching the graphs of a collection of equations that are given in the various forms, for example:

Sketch the graphs of the following lines:

$$y = \frac{3}{2}x + 7$$

$$3x - 2y = -14$$

The line that passes through (4,13) and has slope 1.5

The line that has intercepts (0,7) and (-14/3)

What can we do to enrich the task?

Before digging into “fixes, let’s think about a fundamental question: What do we *really* want students to know?

Here are some of our thoughts.

- We’d like students to be able to produce the equation of a line, given the graph – and we’d like them to be able to do so strategically. If they’re given a graph that passes through a whole-number y-intercept, will they use that information? If the graph passes through two points with whole-number coordinates, will they take advantage of that fact?
- Do they know that any two pieces of information determine a line, and that any line that is not parallel to either axis can be written in every single form given above?

Can we challenge them in those directions?

In the interests of time, I’ll highlight some of the opportunities we saw.

On Target 1, we saw these opportunities among others:

- Activity elicits a connection between a method and the underlying concept
- Two ways of looking at a situation are linked mathematically
- Activity provides opportunities for strategic thinking

On Target 2:

- Students compare and contrast alternative methods
- Students reference underlying concepts to justify the validity and applicability of the methods they choose
- Students justify their reasoning to each other

And on Target 3:

- Students work strategically
- Students are working on supporting their ideas with evidence

Any ideas about how to modify the task?

Some of our thoughts...

There's a large space of possibilities here! But, given the goals we outlined above, and some of the opportunities highlighted in the targets, we might consider the following:

- Giving the students a linear graph and asking them in groups to determine the equation of the graph; asking if there is more than one solution, and having the students compare and contrast solutions; and having the students argue about which approach they find "easiest" or "more efficient." Then, is that always the case? What about a different graph? (For problems that can be approached in more than one way, asking students to compare and contrast their approaches can open up very interesting conversations.)
- Asking students to determine if three points are on the same straight line, and to justify their method. This requires looking closely at slope and explores the reason that the point-slope formula works. (See <https://www.map.mathshell.org/lessons.php?unit=8215&collection=8>. This sample lesson addresses many of the opportunities highlighted above.)

3. A secondary grades example - Exponential Decay.

Here is a typical exponential decay problem:

Over the past few years, the number of students enrolled in after-school programs has been decreasing. Each year there is a 11% decrease in student enrollment.

Currently, 13,145 students are enrolled. If this trend continues, how many students will be enrolled in 6 years?

The solution offered is a purely mechanical plug-in to the formula:

$$FV = PV(1-d)^n, \text{ where}$$

FV= future value; PV= present value; d= rate of decay;

and n = number of periods.

There are many exercises of this type. What can we do to enrich them?

As before, Identifying the opportunities:

On Target 1, we noted these opportunities:

- The lesson pulls together previous strands of content
- A concept is applied in a real world context
- Situations are mathematized in ways that give meaning to symbols

On Target 2,

- Students compare and contrast different approaches to a problem
- Students work problems that extend what they know
- Students observe/derive properties of mathematical objects

And on Target 3,

- Students discuss and evaluate each other's approaches and ideas
- Students build and evaluate models of real world situations
- Students look for patterns, making and testing conjectures

Any ideas about how to modify the task?

Rather than work through these issues hypothetically, we can point to a real life example. Alyssa Sayavedra, one of the authors of *On Target*, wanted to achieve precisely the kinds of opportunities described in the previous slide.

She created a "contextual problem" in which students were supposed to use what they knew to build a model of what happened in a realistic scenario – one in which students could use their real-world knowledge while pursuing meaningful mathematics. Here is the task:

Anay buys a car for \$5,000. The car loses 15% of its value every year.

- a. How much is the car worth after 1 year?
- b. Write an equation to model the value of the car over time.
Before you go on, find a way to check/justify that your equation is realistic and show your work.
- c. How long before the car is worth half of its original value?
- d. After owning the car for 10 years, it breaks down. Anay finds out that she will need to replace the clutch to be able to drive the car again. Is it worth it?

Read about what happened in our case study book!

The work we've done so far gives the flavor of *On Target*.

But it's just a beginning: we've only done dimension 1, the mathematics.

There's a lot more in the other 4 dimensions.
Here's a quick peek.

Cognitive
Demand

To what extent are students supported in grappling with and making sense of mathematical concepts?

Core Questions:

1. **Grappling with the mathematics.** What opportunities do students have to grapple with and make their own sense of mathematical ideas in this lesson?
2. **Challenges and productive struggle.** What challenges do students experience with the tasks and activities? What happens when students experience challenges? How does struggle with mathematical ideas support their engagement and understanding?
3. **Supporting engagement.** In what ways does the environment support active engagement and sense making?

**Equitable Access
to Mathematics**

To what extent are all students provided opportunities to engage with the core content and practices of the lesson?

Core Questions:

- 1. In what ways do all students have opportunities to engage with the core mathematical content of the lesson?**
- 2. In what ways are specific student needs addressed during classroom instruction?**
- 3. In what ways are diverse student strengths leveraged during classroom instruction?**

**Agency, Ownership
and Identity**

How do classroom activities invite students to connect to mathematics – to explore, to conjecture, to reason, to explain and to build on emerging ideas, helping them develop agency, personal ownership of the content, and positive disciplinary identities?

Core Questions:

- 1. How do classroom activities help students connect their personal identities with their mathematical experiences, so they can see themselves as mathematical sense makers?**
- 2. In what ways do classroom activities support students in developing the disposition and capacity to engage with rich mathematical ideas?**
- 3. In what ways do classroom activities provide opportunities for students to make the mathematics their own?**

Formative
Assessment

To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address emerging misunderstandings?

Core Questions:

- 1. In what ways are student ideas, strategies and reasoning processes brought out into the open?**
- 2. In what ways are students' informal understandings and language use valued and built on?**
- 3. In what ways do whole class activities or group interactions support the refinement of student thinking?**

Working in Partnership With Schools

For me, the most important idea is that of the school (or district) being a learning community.

Learning is a long-term process. The goal is gradual improvement over time, through reflection (and helpful tools).

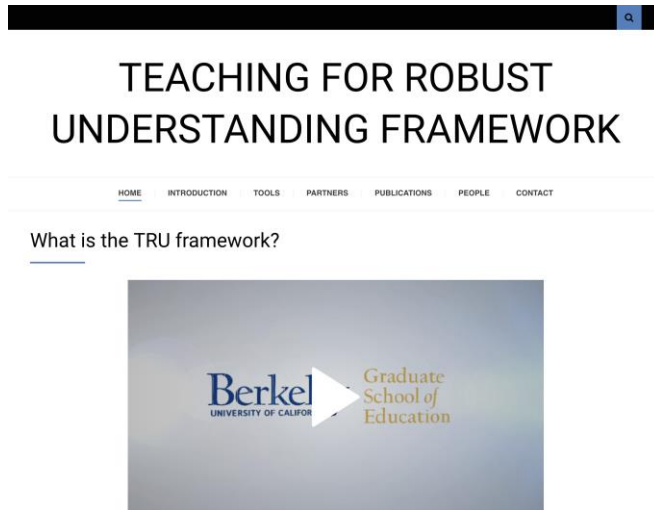
The idea of community is fundamentally important. We learn best when we have trusted colleagues to exchange ideas – and observations! – with.

Any facilitator is a guest who has to respect and work with that community.

There's a lot more to dig into, I'm sorry we have so little time.

I hope you find the ideas worthwhile, and I welcome your questions and comments.

Here's our website, which will have updates on these ideas and more: [TRUframework.org](https://truframework.org)



THANKS!

